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Optimal GENCO bidding strategy

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

Electricity industries worldwide are undergoing a period of profound upheaval. The conventional vertically integrated mechanism is being replaced by a competitive market environment. Generation companies have incentives to apply novel technologies to lower production costs, for example: Combined Cycle units. Economic dispatch with Combined Cycle units becomes a non-convex optimization problem, which is difficult if not impossible to solve by conventional methods. Several techniques are proposed here: Mixed Integer Linear Programming, a hybrid method, as well as Evolutionary Algorithms. Evolutionary Algorithms share a common mechanism, stochastic searching per generation. The stochastic property makes evolutionary algorithms robust and adaptive enough to solve a non-convex optimization problem. This research implements GA, EP, and PS algorithms for economic dispatch with Combined Cycle units, and makes a comparison with classical Mixed Integer Linear Programming.

The electricity market equilibrium model not only helps Independent System Operator/Regulator analyze market performance and market power, but also provides Market Participants the ability to build optimal bidding strategies based on Microeconomics analysis. Supply Function Equilibrium (SFE) is attractive compared to traditional models. This research identifies a proper SFE model, which can be applied to a multiple period situation. The equilibrium condition using discrete time optimal control is then developed for fuel resource constraints. Finally, the research discusses the issues of multiple equilibria and mixed strategies, which are caused by the transmission network. Additionally, an advantage of the proposed model for merchant transmission planning is discussed.

A market simulator is a valuable training and evaluation tool to assist sellers, buyers, and regulators to understand market performance and make better decisions. A traditional optimization model may not be enough to consider the distributed, large-scale, and complex energy market. This research compares the performance and searching paths of different artificial life techniques such as Genetic Algorithm (GA), Evolutionary Programming (EP), and Particle Swarm (PS), and look for a proper method to emulate Generation Companies' (GENCOs) bidding strategies.

After deregulation, GENCOs face risk and uncertainty associated with the fast-changing market environment. A profit-based bidding decision support system is critical for GENCOs to keep a competitive position in the new environment. Most past research do not pay special attention to the piecewise staircase characteristic of generator offer curves. This research proposes an optimal bidding strategy based on Parametric Linear Programming. The proposed algorithm is able to handle actual piecewise staircase energy offer curves. The proposed method is then extended to incorporate incomplete information based on Decision Analysis. Finally, the author develops an optimal bidding tool (GenBidding) and applies it to the RTS96 test system.

CHAPTER 1. OVERVIEW

1.1 Electric Power Industry Deregulation

The electric power industry started over 100 years ago with the electrical pioneers of the late 1800s. For decades, the electric industry is viewed as a benchmark of natural monopoly where economies of scale overwhelm the deadweight losses associated with monopoly operation, for example, a high-capacity generator is less expensive to build and operate than two small generators; a set of transmission or distribution lines to serve customers down a street is less expensive than two parallel ones [1]. Electric power generation, transmission, and distribution are often carried out within the domain of large vertically integrated utilities. Such a utility is guaranteed to be the only provider in a given service territory by franchise rights. In return, the utility has the obligation to serve everyone within its region [1]. The operation of a vertically integrated utility is shown in Fig. 1. The price setting is done by an external regulatory agency and reflects an average cost incurred in generation, transmission, and distribution. Not every utility is considered as a vertically integrated utility. For example, in the USA there are over 3000 utilities, 30% of which generate power and 70% are distribution utilities that purchase wholesale power then resell to local consumers [2]. The utilities in the USA can be categorized into four classes:

Federally-Owned Utilities: Tennessee Valley Authority (TVA), Bonneville Power Administration (BPA);

Investor-Owned Utilities (IOU): PG&E, TXU, MidAmerican, Alliant Energy;

Other Publicly-Owned Utilities: Ames Electric Power Plant (Municipal);

Cooperatively-Owned Utilities: Rural (Co-Gen);

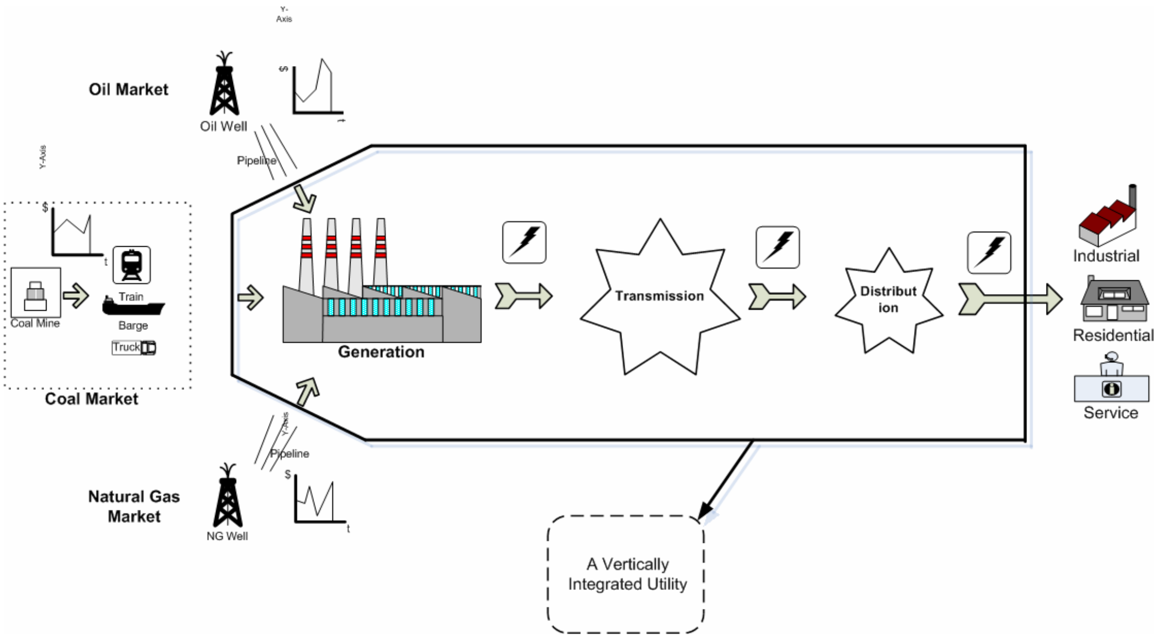

Figure 1: A Regulated Electric Power Industry

During the 1990s, the economic incentives to pursue cheap and reliable electric power supply have forced vertically integrated utilities worldwide to change their ways of doing business from regulation to deregulation. The generation and distribution (or consumption) sectors are divested from transmission. Competition and commercial incentives are introduced into generation and consumption segments, while the transmission part remains regulated and "open-access" to everyone. The idea of “deregulation” is widely believed to contribute a more efficient and economical electrical power industry in today’s environment. The motivations for deregulation are many and differ over regions and countries [3].

First, monopolistic inefficiency, which is a prominent cause to high energy price, increase distrusts of the public and investors for regulation. This has affected the availability of financial investments in expanding generation and transmission capacities. In such situations, many countries restructure their power sectors in order to enhance efficiency, lower price, and provide better service.

Second, economies of scale in the generation sector began to point downward with the advances in generation technology. The efficiency of a traditional fossil fuel unit increases with its rating only if capacity belongs to a certain range. Moreover, combined cycle units and renewable power plants (such as wind, solar, and biomass etc.), which are very popular now, are of higher efficiency, faster response speed, and environmental friendliness. However, they tend to be lower in rating because of design complexities. Besides, two major reasons make small units and distributed generations attractive to utilities and independent power producers [4, 5]:

Smaller plants can be built more quickly, and their construction costs are consequently subject to less economic uncertainty.

Smaller plants can be located more closely to load centers, an attribute that decreases system losses and tends to be advantageous for system reliability.

Table 1: Relative Size of Generation Units [2]

Unit type	Capacity
Nuclear	$\geq$ 1000 MW
Hydro	100 – 200 MW
Fossil steam	200 – 800 MW
Gas turbine	$\leq$ 200 MW
Wind turbine	400 KW – 1.2 MW
Solar and fuel cell	100 KW

Third, the development of Computer Science makes people believe that a competitive electricity market can be implemented and operated while maintaining power system security. Before deregulation, it was felt that the coordination required in operating a power system precluded competition among its participants [4]. However, the progress of both hardware and software makes coordination possible in a competitive environment. Modern computer networks and database techniques provide a solid hardware foundation for market data exchange and information storage. Novel decision support software packages are able to assist participants to play rationally in a competitive and fast-changing marketplace.

Fourth, in 1988 Dr. Fred Schweppe of MIT firstly outlined a plausible method [6], named spot pricing, by which electric energy could be bought, sold, and traded in real time at marginal costs, and those costs take into account time- and space– varying values of electricity. The articulation of how an electric energy marketplace might operate enabled competition in electric energy to be seriously considered [4].

Fifth, several industries such as Natural Gas, Communications, and Airlines have been deregulated recently, and these successful examples provide good models for the electric power industry [1].

A variety of models for deregulation have been proposed, investigated, and implemented. A common strategy is that regulatory agencies re-constructs the electric power industry by breaking vertically integrated utilities into horizontally independent entities including Generation Company (GENCO), Transmission Company (TRANSCO), Distribution Company (DISCO), and Load Serve Entity (LSE) among other possibilities.

The institutional divesture of generation and distribution from transmission lays down the economic foundation of current electricity markets. Fig. 2 shows a typical structure of a deregulated electric power industry. A non-profit entity Independent System Operators (ISO) is established which has the responsibility of ensuring the reliability and security of system operation [7]. The Federal Energy Regulatory Commission (FERC) proposed Standard Market Design (SMD) to unify best practices in market design in 2002. Since it is impossible to recommend a solution that fits all situations, the SMD allows for regional variations. Therefore, these responsibilities may vary widely among the different ISOs existing or emerging in the U.S. and other countries.

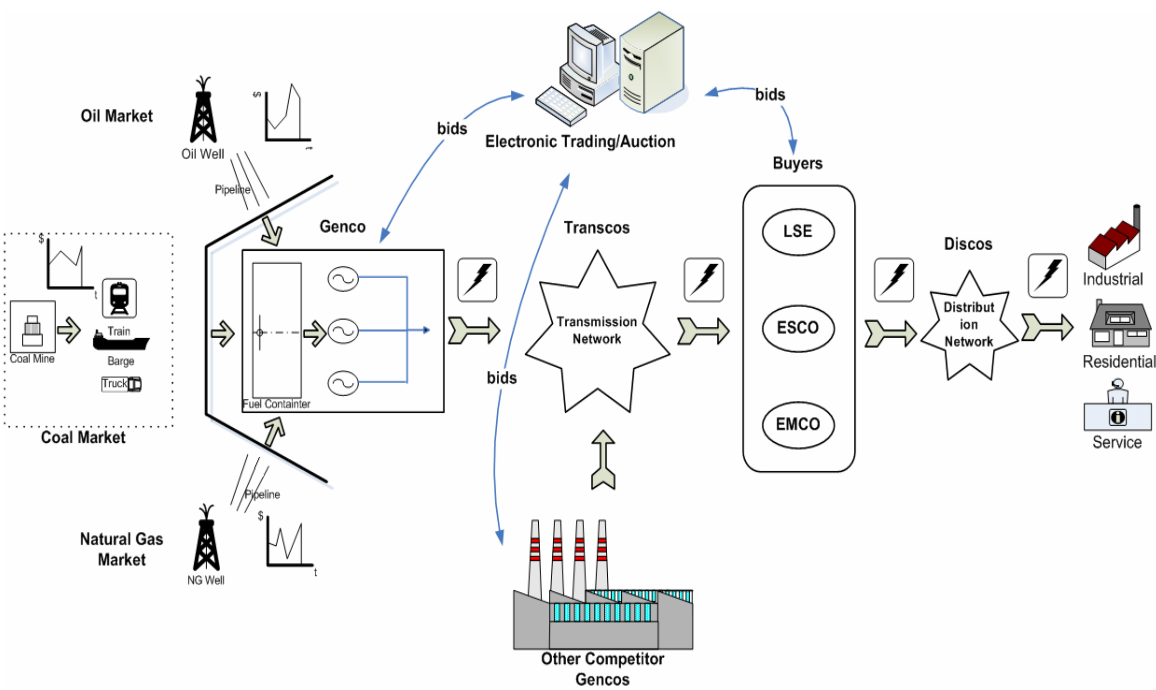

Figure 2: A Deregulated Electric Power Industry

On December 20, 1999, the Federal Energy Regulatory Commission (FERC) issued Order No. 2000, which is intended to promote efficient, reliable, non-discriminatory transmission systems based on market mechanisms. Under FERC Order No. 2000, Regional

Transmission Organizations (RTOs) are being created to operate regional transmission systems and satisfy eight minimum functions [8]. Fig. 3 shows some existing and proposed RTOs in North America until March, 2007. [9]

Tariff design and administration

Transmission congestion management

Management of parallel-path/loop flow

Procurement/provision of ancillary services necessary for grid operations

OASIS administration, including identification of total transfer capability (TTC) and available transfer capability (ATC)

Market monitoring

Transmission planning and expansion

Interregional coordination

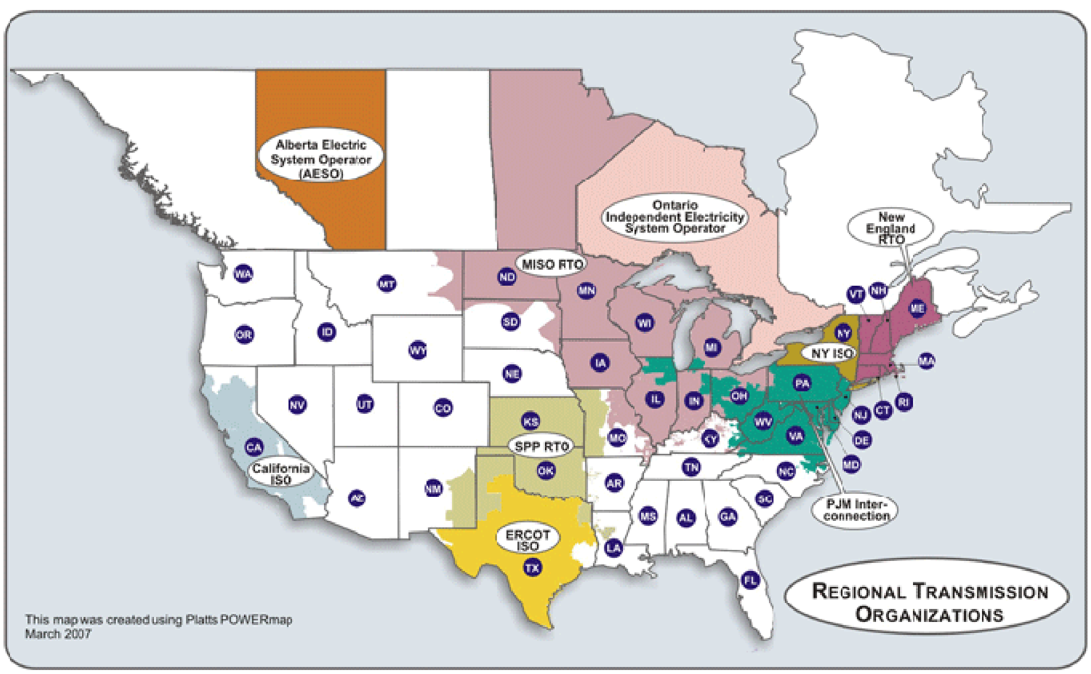

Figure 3: Existing and Proposed RTO Configurations [9]

1.2 Organization of the Dissertation

This dissertation is organized in seven parts. Chapter One introduces the deregulation of electric power industry and this work. Chapter Two first reviews the past research done on electricity market simulation. Since this research mainly focuses on GENCOs' behaviors, not other players, suppliers' bidding strategies and supply function equilibrium are reviewed and discussed in detail. Furthermore, evolutionary algorithms are introduced and summarized in section 2.4. Chapter Three focuses on a basic economic dispatch problem that can be regarded as a prototype of bidding where units naively provide true information. But the emphasis is to involve non-convex cost functions. Chapter Four develops two supply function equilibrium bidding models: one is to involve GENCOs' inter-temporal production constraints and the other is to consider transmission congestion. Chapter Five compares the performance of learning algorithms for EDC and explores the feasibility for GENCOs to evolve the market equilibria. Chapter Six focuses on a classical optimization method ---- Linear Programming, to study optimal bidding strategies. The proposed method considers the non-convexity of piecewise staircase energy offer curve. It also incorporates incomplete information based on Decision Analysis. Finally, the author develops an optimal bidding tool and applies it to the RTS 96 test system. Chapter Seven summarizes this dissertation and makes a conclusion.

CHAPTER 2. LITERATURE REVIEW

2.1 Electricity Market Simulation

Electric power industry worldwide is undergoing a period of profound upheaval. Conventional vertically integrated mechanism is replaced by a competitive market environment. A pure operating cost optimization is not enough to capture all of characteristics of the distributed, large-scale, and complex system. A market simulator will be a valuable training and evaluation tool to assist sellers, buyers, regulators, and other players to understand market's dynamic performance, price fluctuation, and make better decisions avoiding bulk risks in both short-run operation and long-run planning [10].

A software package called production-costing program was developed before deregulation and typically was designed to handle large number of generation units operating under a centralized utility. This program calculates a generation systems' production costs (costs of generating power) together with generation reliability indices for long-run generation planning decisions. Such programs incorporate fuel costs and heat rates together with probabilistic models of the load and each generator's availability. Because of the computational requirements to handle the probabilistic nature of the models (convolution), such programs usually do not represent the network [4].

A market simulator can be regarded as a modern version of the production-costing program. This simulator not only keeps the function to track a generation system's optimal production decision, but also models every other player's behavior and the rules of interaction among players at both of physical and economic layers. Market state variables such as market clearing price (MCP) / locational marginal price (LMP) etc. evolve with respect to time according to the interactions between market participants. Some critical insights like market efficiency, transmission congestion effects as well as market power can be also achieved through a simulation process.

To date there have been a number of market simulators proposed and implemented in literatures. The simulators can be classified in terms of the market structure, the representation of transmission network, and the modeling of interaction between players etc.

Market Structure

Three common market structures have been assumed in proposed market simulators. A power pool is a centralized marketplace where market participants submit price/quantity bids/offers. A market operator organizes and regulates the bids/offers by an auction mechanism. If both GENCOs and DISCOs can compete, it is called a double side auction. If only supplier can compete in the pool, it is called a single side auction, which happens when demand side is of less elasticity. Two settlement mechanisms in pool are proposed – uniform pricing (all accepted suppliers are charged with the last accepted bid) and pay-as-bid (all accepted suppliers are charged with their own bids) [1]. Bilateral contracts are negotiable agreements between sellers and buyers (or traders) about power supply and receipt. The bilateral-contract is flexible and can be signed Over-the-Counter (OTC); negotiating parties can specify their own contract terms [11]. The hybrid model combines features of pools and bilateral contracts. In this model, a pool isn't mandatory, and customers can either negotiate a power supply agreement directly with suppliers or accept power at the pool market price. This model therefore offers more customer choices [11].

Transmission Network

Because of highly nonlinearity, some market simulators disregards transmission network entirely [12] [13] [14]. It is as if all of buyers and sellers are connected at a single physical bus/hub or transmission network has infinite transfer capacity [4]. The simplified model brings in huge conveniences for studying pure economic interaction at the cost of losing some accuracy and price predictability. Sheble etc. [15] presented and implemented a market simulator (Market Sim) based on a transportation model. The network capacity is represented by available transfer capability (ATC). ATC value is updated after each round simulation. Yang [16], and Torre [17, 18], used DC power flow to represent transmission network in market simulators. Some algorithms like linear programming (LP) or mixed integer programming (MIP) can provide on-line sensitivity analysis. However, DC power flow is not able to handle reactive power and voltage stability issues. One tool (MAPS) developed by GE uses a detailed electrical model of the entire transmission network, along with generation shift factors determined from a solved AC load flow, to calculate real power flows. This tool captures the economic penalties of re-dispatching generation to satisfy transmission line flow limits and security constraints [4].

Modeling of Interaction

There are at least two distinct approaches to model interaction between players. First, equilibrium modeling uses a simplified mathematical representation of a market. J. Nash described the elementary principle of an equilibrium as a condition where no player can improve his situation by playing differently [19]. Two famous Oligopolistic models in a market for homogeneous products are the Cournot and Bertrand models [20]. In the Cournot model, players choose their levels of production assuming their rivals do not change production levels, and in the Bertrand model players choose their offer prices assuming their rivals do not change prices. In both cases, the choice of quantity or price will result in the other being uniquely determined. Under simplifying assumptions about costs and demand, it can be proved that equilibrium always exists [19]. The Supply Function Equilibrium (SFE) model is an extension to the Cournot and Bertrand by introducing a functional form to represent variation or uncertainty. Especially players can choose a linear bid/offer function with two parameters, slope and intercept, that holds regardless of the outcome of the demand level. The equilibrium result is distributions for prices and schedules [19]. SFE model will be discussed later in detail.

Next approach is inspired from Experimental Economics that uses laboratory experiments to evaluate economic theory. In power market simulation, computer agents normally are meant to replace human agents, making bid decisions to maximize profits [19]. The term "agent" is usually used to describe self – contained program, which can control their own actions, based on their perceptions of their operating environment [19]. Usually agents have some of these characteristics, (a) Autonomous (b) Intelligent (c) Rational (d) Learning ability (e) Social incorporation ability [21]. Two advantages of computer agents compared to human agents are that their decision-making environment can be precisely controlled and analyzed and that such experiments can be run parametrically in the thousands with little added expense. A disadvantage is that it can be difficult to "teach" computer agents to make complex decisions [19]. Furthermore multi-agent systems (MAS) model complex distributed systems as a set of software agents that interact in a common environment. The integration of a system from a number of agents lets the system react and adapt better in a changing environment [10].

Table 2-4 show some major results from an on-line survey on current research and development of electricity market simulator. This survey is conducted by EPRI [19]

Table 2: Use and Capabilities [19]

	Simulation Type			Problem Size					
Product Name	**Human Experiments**	**Agent-Based Experiments**	**Equilibrium Computations**	**Buses**	**Transmission Lines**	**Generating Units**	**Participants**	**Interdependent Time Periods**	**Interdependent Contingencies**
MAPS	-9	-9	-9	50000	100000	7500	175	8760	5000
EP	1	1	1	limited by solution time		0	0	0	0
MELBOURNE	0	1	0	100	0	100	100	100000	10
GE MAPS	0	0	0	0	0	0	0	0	0
EMCAS	1	1	1	2000	2400	400	?	8760	?
PLEXOS	0	0	1	15000	18000	2000	1000	800000	100000
MADERE	0	1	0	?	?	?	?	1000	1
EE	1	0	1	4	5	40	16	?	?
ENERGY 2020	1	1	1	110	2000	50000	80	500	150
NetaSim	0	0	1	0	0	200	30	0	0
TSCM	0	0	1	53	71	19	5	2	6
COMPETES	0	0	1	~20	~40	~1000	20	1	1
CTCEM	0	0	1	~100	~800	~3000	~800	1	0
COMPETES	0	0	1	19	34	261	10	1	1
PowerACE	0	1	0	1	1	300	100	10000	0
LTEPM	0	0	1	1	0	500	1	14400	5
Eureca	0	0	1	30	30	15000	1	14400	5
PD EMPS Price Forecast	0	0	1	30	30	15000	1	14400	5
IPSPE Model	0	0	1	8	20	1500	0	672	1
STEMS-RT	0	0	1	180	200	200	50	1	4
Market Sim	0	0	1	2500	5000	500	500	8000	2000
GENERIS	0	1	0						
POWERS	0	0	1						

Table 3: Market Participant Modeling Features [19]

	Market Participant Modeling						
Product Name	**Cournot**	**Bertrand**	**Supply Function Equilibrium (SFE)**	**MPEC**	**Heuristic**	**Other**	**Describes**
MAPS	0	0	1	0	1	-9	
EP							
MELBOURNE	1	1	0	0	1	-9	
GE MAPS							
EMCAS	1	1	1	0	1	1	Learning and adaptation
PLEXOS	1	1	0	0	1	1	LRMC Recovery
MADERE	0	0	0	0	1	-9	
EE	-9	-9	1	-9	-9	-9	
ENERGY 2020	0	0	0	-9	1	-9	
NetaSim	-9	-9	-9	-9	-9	-9	
TSCM	1	0	0	1	0	1	EPEC, Subgame Perfect Two Stage Nash
COMPETES	1	1	0	0	0	1	Conjectured Supply Functions
CTCEM	1	1	0	0	0	0	
COMPETES							
PowerACE	0	0	0	0	0	-9	
LTEPM	0	0	1	0	1	-9	
Eureca	0	0	1	1	0	-9	
PD EMPS Price Forecast	0	0	1	1	0	-9	Water Value Method - Multi Area
IPSPE Model	-9	-9	-9	-9	-9	-9	
STEMS-RT	0	0	0	1	1	0	
Market Sim							
GENERIS							
POWERS							

Table 4: Demand Response and Load Modeling [19]

	Demand Response						Load Modeling			
Product Name	**Is Demand Price Responsive?**	**"Hard-wired" Elasticities**	**User-Specified Elasticities**	**User-Specified Reductions**	**Other**	**Wholesale Price Caps**	**Fixed**	**Stochastic**	**Price Responsive**	**Other**
MAPS	1	-9	1	1	-9	1	1	0	1	-9
EP										

Product Name	Demand Response: Is Demand Price Responsive?	Demand Response: "Hard-wired" Elasticities	Demand Response: User-Specified Elasticities	Demand Response: User-Specified Reductions	Demand Response: Other	Wholesale Price Caps	Load Modeling: Fixed	Load Modeling: Stochastic	Load Modeling: Price Responsive	Load Modeling: Other
MELBOURNE	0	-9	-9	-9	-9	1	1	1	0	-9
GE MAPS										
EMCAS	1	1	1	1	-9	1	1	1	1	-9
PLEXOS	1	-9	1	1	1	1	1	1	1	-9
MADERE	1	-9	-9	-9	1	0	1	1	1	-9
EE	1	-9	1	-9	-9	1	1	1	1	-9
ENERGY 2020	1	-9	-9	-9	1	1	1	1	1	-9
NetaSim										
TSCM	1	-9	1	-9	-9	1	0	0	1	1
COMPETES	1	1	1	-9	-9	0	1	1	1	-9
CTCEM	1	-9	1	-9	-9	0	1	-9	1	-9
COMPETES	1	-9	1	-9	-9	0	1	0	1	-9
PowerACE	1	-9	1	-9	-9	1	1	1	0	-9
LTEPM	0	-9	-9	-9	-9	1	1	0	0	-9
Eureca	0	-9	-9	-9	-9	1	1	0	0	-9
PD EMPS Price Forecast	0	-9	-9	-9	-9	1	1	0	0	-9
IPSPE Model	0	-9	-9	-9	-9	0	1	1	-9	-9
STEMS-RT	1	-9	-9	1	-9	1	1	0	1	-9
Market Sim										
GENERIS										
POWERS										

2.2 GENCO Bidding Strategies

In US, pre-deregulation the electricity whole price is set and regulated by FERC or State Public Utility Commission (SPUC). A vertical integrated electric utility only operates based on cost minimization in a short run, and keeps a rate of return supervised by FERC or SPUC in a long run.

Current industry structure generally requires separating the functions associated with selling and buying electric energy, the generation and distribution (or consumption), from transmission. Market participants have to face the volatility of price and make sure profitable in a long run. Instead of discussing all of players, this paper will mainly focus on the short-run operation strategy of GENCOs. The traditional economic dispatch (EDC) and unit commitment (UC) programs used by electric utilities for many years are only helpful to GENCOs who own multiple generation facilities when they make one offer to the market and then need to dispatch their units in the most economic fashion to deliver this offer. A profit based bidding decision support system is critical for GENCOs to operate in the new environment.

The previous research on bidding strategies is methodologically classified into the following three groups.

Pure Optimization Model

The first group of research pays attention to a specific player, the one under study. The idea is to simplify "the rest of the world" as a set of exogenous variables (stochastic or deterministic). The group of study has developed many mathematical programming models to find an optimal bidding strategy (e.g. Dynamic Programming, Fuzzy Linear Programming, and Stochastic Dynamic Programming etc.). A bidding strategy using Markov Decision Process (MDP) is proposed in [22]. The authors discussed the impacts of production limit and market share on optimal bidding strategies. The number of states is reduced by classifying peak/off-peak load, peak/off-peak price. A decision aid for scheduling and hedging (DASH) model is proposed in [79] for power portfolio optimization. The inputs of the model, electricity demand, electricity forward price, gas forward price, and electricity spot price are captured by several stochastic processes. A multiple time scale decision making problem is solved considering both long-term financial and short-term operational constraints. The group of models is usually easier to generalize and analyze because of well-established mathematical foundation. The disadvantage is that the methods do not model the behavior aspect of players.

Game Theory Model

The second group discussed the bidding strategies from a perspective of players' behaviors. It also can be called equilibrium model for the whole purpose of the group is to find economic equilibria of the system. The mutual interaction is represented by Game Theory. Game Theory can be classified into two areas – Cooperative and Non-cooperative [20]. Cooperative games can be applied to investigate the effects of firms' collusion. In a model named Stackelberg game, a firm as a leader (first-mover) with largest market power is assumed to can manipulate prices subject to the accurately predicted reactions of naïve followers who have small market shares and believe they can not affect prices [23]. The Stackelberg game is also modeled as a mathematical programming with equilibrium constraints (MPEC) problem [24].

In more competitive models, Cournot and Bertrand are assumed to represent the type of interaction [25] [26] [27] [28] [29]. However, assumptions of the two models are naïve for ISO-type auctions, in which GENCOs bid a whole set of price/quantity pairs for each generator. In this case, decision variables become parameters of a function that determines a relationship between price and quantity that GENCO is willing to produce. The type of competition is termed Supply Function Equilibrium. Next section will review Supply Function Equilibrium model in detail. Game with incomplete information is discussed in [30]. A contribution of this group is that this research provides analytical rationale and explanation regarding how market power can be exercised by strategic bidding behavior [31]. However, game theory itself is based on the rationality of all players. This assumption does not hold in practice. The issue of multiple equilibria frustrates a lot of game theorists. This type of game model is logically limited lack of dynamic theory, too.

Agent and Heuristic Model

The last group of past research efforts fully utilizes computer science methods to mimic human beings and simulate optimal bidding strategies [31]. Sheble [13] etc. proposed a genetic algorithm based framework to evolve utility bidding strategies in a double side auction marketplace and developed a market simulator by Pascal language. An evolutionary programming bidding strategy is discussed in [32]. F. F. Wu etc. [33, 34] etc. have discussed those issues on modeling electricity market as MAS, both theoretical and practical aspects. Reinforcement learning algorithms are considered to “teach” an agent to learn the optimal bidding strategy. A particular reinforcement-learning algorithm–Roth/Erv algorithm is implemented in [35] [36]. Others joined MDP model in Group 1 with some reinforcement learning algorithms. They proposed to use Q-learning or Actor/Critic learning algorithms to solve optimal policy for a MDP with incomplete information [37] [38] [39]. The basic intuition underlying reinforcement learning is that the tendency to implement an action should be strengthened if it produces favorable results and weakened if it produces unfavorable results [40]. The group of models is more flexible, robust, and easily implemented compared with analytical (mathematical) approach (Group 1 and 2). However, the drawback is that the underlying mathematical foundation is not well developed.

2.3 Supply Function Equilibrium Model

The general supply function equilibrium (SFE) model was introduced by Klemperer and Meyer (1989) and applied by Green and Newbery (1992) to the electricity industry reform in England and Wales (E&W). SFE is attractive compared to Cournot or Bertrand because it offers a more realistic view of electricity market, where bidding rules typically require suppliers to offer a price schedule (a supply curve) rather than a series of pure quantity or pure price bids. [41]

In SFE model, functional forms must be specified for demand, cost, and supply. A particularly simple form is to assume a linear demand function, a quadratic convex cost function, and a linear supply function. The great advantage of the SFE with linear functional forms over the more general form is the analytical solvability.

Assume that for GENCO *i,* the true cost function is given by a quadratic convex function $Cost_i(q_{it}) = \alpha_i q_{it} + 0.5\beta_i q_{it}^2, \quad \beta_i > 0$ (1)

It is also assumed that the rules require GENCO i to bid a linear increasing supply function with two strategic parameters: intercept l_{it} and slope k_{it}.

$$P_t = l_{it} + k_{it} q_{it}, \quad k_{it} > 0 \tag{2}$$

The system integrated demand curve is assumed to be

$$P_t = h_t - g_t Q_t, \quad g_t > 0 \tag{3}$$

The market clearing condition is

$$Q_t = \sum_{i=1}^{I} q_{it} \tag{4}$$

In this stage, transmission network impacts are not considered yet. Therefore, the market clearing price is the same for all players, and this market clearing condition maximizes the social welfare.

The profit for GENCO i at time t is

$$\pi_{it} = P_t q_{it}(P_t) - Cost_i(q_{it}) \tag{5}$$

Each GENCO i manipulates its parameters, intercept l_{it} and slope k_{it}, to maximize its profits, while the market clearing condition is satisfied. The supply function equilibrium implies no player can increase profit by unilaterally change its bid supply function. [42]

In Mathematics, a strategies profile $\{l_{it}^* \& k_{it}^*, \text{ for all } i = 1,...,I,\ t = 1,..,T\}$ is called to constitute a Supply Function Nash Equilibrium, if, with this strategic parameters $l_{it}^* \& k_{it}^*$, GENCO i maximizes his profit $\pi_{it}\left(l_{it}, l_{-it}, k_{it}, k_{-it}\right)$, given all of other GENCOs stick to the bid $l_{-it}^* \& k_{-it}^*$. The reasoning must hold for $\forall i \in I$.

The optimal condition of SFE is to set the first-order derivative to be zero for all of GENCOs:

$$\frac{d\pi_{it}}{dP_t} = q_{it}(P_t) + \left[P_t - \frac{dCost_i(q_{it})}{dq_{it}}\right]\frac{dq_{it}(P_t)}{dP_t} = 0,\ \ for\ \forall\ i \tag{6}$$

Using the market clearing condition (4), one obtains

$$q_{it}(P_t) = \left[P_t - \frac{dCost_i(q_{it})}{dq_{it}}\right]\left(-\frac{dQ_t}{dP_t} + \sum_{v=1, v\neq i}^{I} \frac{dq_{vt}(P_t)}{dP_t}\right) = 0,\ \ for\ \forall\ i \tag{7}$$

The basic equation governing the SFE solution is provided by Green (1996). Any solution to these coupled differential equations such that each q_i is non-decreasing over the relevant range of prices is an SFE [42]. However, the SFE may not exist since it is a nonlinear differential equation system.

Substituting (1), (2), and (3) into equation (7), one obtains

$$\frac{P_t - l_{it}}{k_{it}} = \left(P_t - \alpha_i - \beta_i \frac{P_t - l_{it}}{k_{it}} \right) \left(\frac{1}{g_t} + \sum_{v=1, v \neq i}^{I} \frac{1}{k_{it}} \right)$$

After some rearrangement, it becomes to be

$$\frac{1}{k_{it}} P_t - \frac{l_{it}}{k_{it}} = \left(\left(1 - \frac{\beta_i}{k_{it}} \right) P_t - \alpha_i + \frac{\beta_i l_{it}}{k_{it}} \right) \left(\frac{1}{g_t} + \sum_{v=1, v \neq i}^{I} \frac{1}{k_{it}} \right), \ for \ \forall \ i \qquad (8)$$

In previous literatures, four alternative specifications of the strategic parameters are summarized by R. Baldick [43]:

l_{it} – parameterization, where player i can choose the intercept arbitrarily in supply function but is required to specify a fixed, pre-chosen slope k_{it}. Usually, the fixed value is further assumed to be equal to the true slope in marginal cost function, β_i.

k_{it} – parameterization, where player i can choose the (non-negative) slope arbitrarily in supply function but is required to specify a fixed, pre-chosen intercept l_{it}. Usually, the fixed value is further assumed to be equal to the true intercept in marginal cost function, α_i.

$(k_{it} \propto l_{it})$ – parameterization, where player i can choose k_{it} and l_{it} subject to the condition that k_{it} and l_{it} have a fixed linear relationship. Usually, the fixed value is further assumed to be equal to the true ratio of slope and intercept in marginal cost function, $\frac{l_{it}}{k_{it}} = \frac{\alpha_{it}}{\beta_{it}}$.

(k_{it}, l_{it}) – parameterization, where play i can choose k_{it} and l_{it} arbitrarily.

According to equation (8), since there are totally I equations and $2I$ unknowns $\{l_{it}, k_{it} \mid i = 1,\ldots,I\}$, it is inevitable that multiple SFEs exist when players have full flexibility in choosing the parameters specifying their bid functions (k_{it}, l_{it}). Therefore, the bottom line is that one has to assign I parameters, then calculate the other I parameters under equilibrium conditions.

In modeling the E&W market, Green and Newbery [42, 44] make an important assumption, which is called Assumption (*):

Assuming each player as specifying a single supply function bid that applied to all pricing periods over an extended length of time. (*)

The assumption was true on a daily basis in the E&W pool until 2001. Actually, the assumption implicitly implies k_{it} – parameterization with l_{it} equal to the true intercept α_i, which can be shown as follows:

Revisiting equation (8), since the bid function is required to be consistent during a period of time, (8) should be satisfied for the realized prices during that time period. If there are at least two levels of demand cleared during that period, then there will be at least two prices realized and (8) will be satisfied as an identity [45]. Consequently, the coefficient of price on the left hand and right hand of (8) should be equal, and similarly the constant term should be equal. Therefore, the conditions for an SFE are:

$$\frac{1}{k_{it}} = \left(1 - \frac{\beta_i}{k_{it}}\right)\left(\frac{1}{g_t} + \sum_{v=1, v\neq i}^{I} \frac{1}{k_{it}}\right) \tag{9}$$

$$\frac{l_{it}}{k_{it}} = \left(\alpha_i - \frac{\beta_i l_{it}}{k_{it}}\right)\left(\frac{1}{g_t} + \sum_{v=1, v\neq i}^{I} \frac{1}{k_{it}}\right) \tag{10}$$

Substituting (9) into (10), one obtains

$l_{it} = \alpha_i \quad for\ \forall\ i$

Conversely, it is also easy to show that k_{it} – parameterization with l_{it} equal to the true intercept α_i, implies Assumption (*).

Therefore, generally speaking,

Assumption (*) $\Leftrightarrow$ k_{it} – parameterization with $l_{it} = \alpha_{it}$, which is defined as the two conditions: $\begin{cases} \dfrac{1}{k_{it}} = \left(1 - \dfrac{\beta_i}{k_{it}}\right)\left(\dfrac{1}{g_t} + \displaystyle\sum_{v=1, v\neq i}^{I} \dfrac{1}{k_{it}}\right) \\ l_{it} = \alpha_{it} \end{cases}$

However, in other markets a different supply function can be specified for each pricing period. Assumption (*) in [42, 44] of a single supply function applying across multiple pricing periods does not hold. Furthermore, assuming the consistency of bid supply functions across multiple periods severely restricts the flexibility of bids compared to the true flexibility in these markets [45]. Consequently, k_{it} – parameterization can not be applied into either a market without the Assumption (*) or multiple period situations.

On the contrary, Hobbs and his colleagues [23] [24] use a single pricing-period model; that is, they do not assume that the supply function must be the same across multiple pricing periods. They argue that GENCOs only manipulate the intercept of the bid functions (*i.e.* l_{it} – parameterization), and not its slope. The reasons they proposed are [24]:

Slopes of marginal cost functions for individual generators are usually shallow, so the very steep slopes that would result from manipulating just k_{it} would not be credible.

The steepness of an aggregate bid curve for an entire firm can be manipulated by having different markups $l_{it} - \alpha_{it}$ for different units.

From my point of view, the main reason is that many electricity markets worldwide allow GENCOs to bid a different supply function for each period. Assumption (*) does not hold and k_{it} – parameterization is not valid. Another key point is that since l_{it} – parameterization leads a linear equation system, the existence and uniqueness of the solution is easy to prove. A lot of computational difficulties can be reduced in l_{it} – parameterization compared to k_{it} – parameterization.

2.4 Evolutionary Computation and Artificial Life Techniques

Evolutionary computation is a general term for several computational techniques that take their inspiration from natural selection in the biological world and use this mechanism of "Evolution" as key elements in their design and implementation [46]. There are a variety of evolutionary computational methods that have been proposed and studied which are referred to as evolutionary algorithms [47]. ALIFE (artificial life) is a common acronym to tie all of the ideas based on biological emulation, including evolutionary computation. It is the attempt to simulate, or in some case emulate, the governing principles of life [47]. Artificial life techniques have found success in solving several different complicated centralized non-convex optimization problems and for emulating intelligent market participants' individual decentralized optimization problem.

Evolutionary algorithms share some common conceptual base of simulating the evolution of individual structures via processes of selection, reproduction, and mutation. Several different types of evolutionary algorithms were developed independently. These include genetic programming (GP), evolutionary programming (EP), evolutionary strategies

(ES) and genetic algorithms (GA) etc. This part of the proposal summarizes the following algorithms: Genetic Algorithms, Evolutionary Programming, and Particle Swarm.

2.4.1 Genetic Algorithm

Genetic algorithm (GA), developed by John Holland (1975), has traditionally used a more domain independent representation, namely, bit-strings. However, many recent applications of GAs have focused on other representations, such as graphs, lisp expressions, ordered lists, and real-valued vectors.

Genetic algorithms are performance driven method of finding useful structures with a computer, based loosely on the theory of evolution. In evolution successful creatures mate, blending their genes, then undergoing a small number random change via mutation. A genetic algorithm uses selection to pick parents in some sort of relation to their quality. It blends structures via a process called crossover. Small changes are accomplished by mutation [48].

Crossover

Crossover selects bits from each of two genes to produces new genes. There are several kinds of options for performing crossover. The "No crossover" makes the new genes copies of the old genes. When this is used, all innovation in the searching results from mutations. One-point crossover chooses a random position in the genes and exchanges the bits after that point. Two-point crossover chooses a pair of positions in the genes and exchanges those bits between the positions. Uniform crossover either swaps or leaves alone the bits in the genes at each position with a probability of 0.5. This form of crossover is probably best for efficient search but is computationally intensive, as it requires a lot of random numbers.

Mutation

Mutation is a mechanism to produce new pieces of genes that may contain new information. The mutation rate is the probability, independent for each bit, that the bit will be flipped. Mutation provides an ongoing source of exploration in the searching. This is absolutely necessary in a situation where the measure of quality is not absolute.

Selection

Normally there are three selection techniques, proportional selection, rank selection, and roulette selection. All of them choose potential parental strategies in proportion to a number derived from the individual's fitness [49].

Proportional selection takes the fitness, multiplies it by a scaling constant (ratio) and then adds a fixed offset. The probability of being chosen as a parent is directly proportional to this number. Proportional selection is used to speed convergence toward a particular value but may cause thrashing when the measure of quality is not absolute.

Rank selection places the individuals in order from *n* (most fitness) to 1 (least fitness) and chooses them in proportion to their rank. This method of selection tends to both reduce the importance of differences between individuals when those differences are large and increases the importance of differences when they are small.

Roulette selection chooses individuals in direct proportion to their fitness. It is equivalent to proportional selection with ratio one and offset zero. It is included as a separate option because of the prominence of roulette selection in the theory of genetic algorithms.

2.4.2 Evolutionary Programming

Evolutionary Programming (EP) is another technique in the field of evolutionary computation. EP was proposed as a Finite State Machine (FSM) model by L. J. Fogel in 1960s. In that model, the mutation operator of the state of machine is a kind of a uniform distribution. In 1990s, the idea of evolutionary programming was extended by D. B. Fogel to optimization. Now, EP has become a powerful optimization tool and has been applied to many real problems [50]. Unlike Newton method, EP does not depend on the curvatures of the objective functions but rather it is based on the mechanics of natural selections. The purpose of evolutionary programming is to do a stochastic search in order to seek an optimal solution to an optimization problem [51].

The schematic diagram of the EP algorithm for optimization is depicted in Fig. 4:

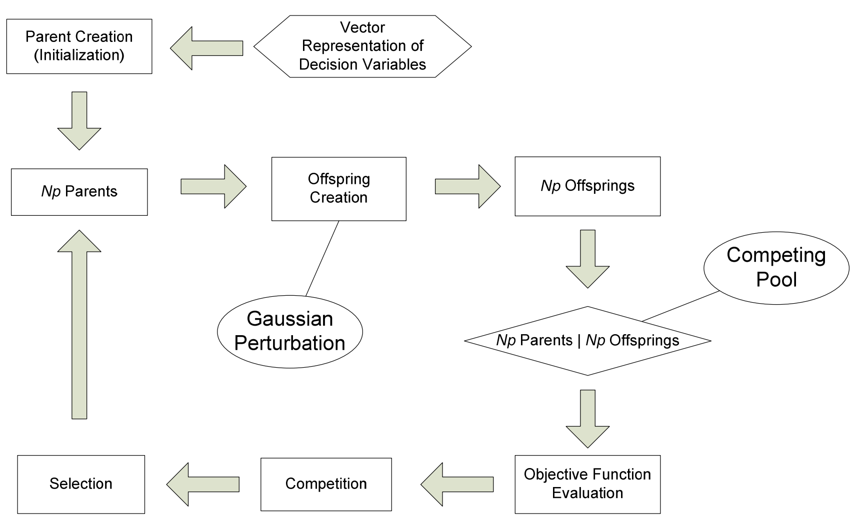

Figure 4: Schematic Diagram of EP algorithm

EP can be represented by a dynamic equation:

$$X_i^{k+1} = X_i^k + Gaussian(0,(\varphi_i^k)^2) \quad (11)$$

Where X_i^k represents individual i at generation k and X_i^{k+1} is individual i at generation k+1.

The standard deviation φ_i^k is the mutation factor and indicates the range the offspring is created around its parent and is given by:

$$\varphi_i^k = \beta\left(\frac{f_{best}^k}{f_i^k}\right) \tag{12}$$

Where:

β : the scaling factor

f_{best}^k : the best fitness at generation k

f_i^k : the fitness of individual i at generation k

The magnitude of φ_i^k has impacts on the magnitude of changing in the offspring compared to the parent. The bigger the φ_i^k, the higher possibility the greater changing in X_i^{k+1} compared to X_i^k and vice versa.

2.4.3 Particle Swarm

Particle Swarm Optimization (PSO) is a recently proposed algorithm by James Kennedy and R. C. Eberhart in 1995, motivated by social behavior of organisms such as bird flocking and fish schooling. PSO algorithm is not only a tool for optimization, but also a tool for representing sociocognition of human and artificial agents, based on principles of social psychology. PSO as an optimization tool provides a population-based searching procedure in which individuals called particles change their position (state) with time. In a

PSO system, particles fly around in a multidimensional searching space. During flight, each particle adjusts its position according to its own experience, and according to the experience of a neighboring particle, making use of the best position encountered by itself and its neighbor. Thus, a PSO system combines local search methods with global search methods, attempting to balance exploration and exploitation [52].

PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the solution space by following the current optimum particles.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. In one iteration, each particle is updated by following two "best" values. The first one is the best solution (fitness) it has achieved so far. (The fitness value is also stored.) This value is called *Pbest*. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called *Gbest*. When a particle takes part of the population as its topological neighbors, the best value is a local best and is called *Lbest*.

Assuming a physical *n*-dimensional searching space, the position and velocity of individual i are represented as the vectors $X_i = (x_{i1}, \ldots, x_{in})$, and $V_i = (v_{i1}, \ldots, v_{in})$, respectively, in the PSO algorithm. Let $Pbest_i = (x_{i1}^{Pbest}, \ldots x_{in}^{Pbest})$, and $Gbest_i = (x_{i1}^{Gbest}, \ldots, x_{in}^{Gbest})$, respectively, be the best position of individual i and its neighbors' best position so far. Using the information, the updated velocity of individual i is modified under the following equation in the PSO algorithm:

$$V_i^{k+1} = \omega V_i^k + c_1 rand_1 * (Pbest_i^k - X_i^k) + c_2 rand_2 * (Gbest_i^k - X_i^k) \quad (13)$$

Where:

V_i^k velocity of individual i at iteration k;
ω weight parameter;
c_1, c_2 weight factors;
$rand_1, rand_2$ random numbers between 0 and 1;
X_i^k position of individual i at iteration k;
$Pbest_i^k$ best positon of individual i until iteration k;
$Gbest_i^k$ best positon of the group until iteration k.

Each individual moves from the current position to the next one by modified velocity (13) using the following equation:

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (14)$$

Particles' velocities on each dimension are clamped to a maximum velocity V_{max}. If the sum of accelerations would cause the velocity on that dimension to exceed V_{max}, which is a parameter specified by the user. Then the velocity on that dimension is limited to V_{max}.

CHAPTER 3. ECONOMIC DISPATCH WITH COMBINED CYCLE UNITS

3.1 Introduction

The economic dispatch (EDC) function allocates total demand among the available generating units to minimize the total cost of generation. This activity can be executed on a minute-by-minute basis at a control center of utilities or Independent System Operator (ISO) in the world [3]. The general procedure is shown in Fig. 6. The system operators know cost of each unit and are able to control generating facilities to produce predetermined quantities. Cost curves of conventional thermal units can be modeled as quadratic convex functions. Equal incremental cost is criteria to solve the traditional EDC problem [53]. Competitive market forces GENCOs to apply novel technologies such as Combined Cycle (CC), Integrated Gasification Combined Cycle (IGCC), Fuel Switching/Blending, Constant/Variable Pressure, Overfire, and Dual Boiler etc. in stead of only thermal units to pursue profits. CC units utilize both gas turbines and steam turbines to produce electrical energy. The waste heat from the combustion turbines is directed into a boiler just as steam from the boiler is used to drive steam turbines. An illustration on CC units is shown in Fig. 5. CC units are of relatively high efficiency, have fast ramp rates and exhibit other beneficial features compared to conventional thermal units [54]. This has enabled the CC units to become the technology of choice for many new power facilities wherever natural gas is affordable.

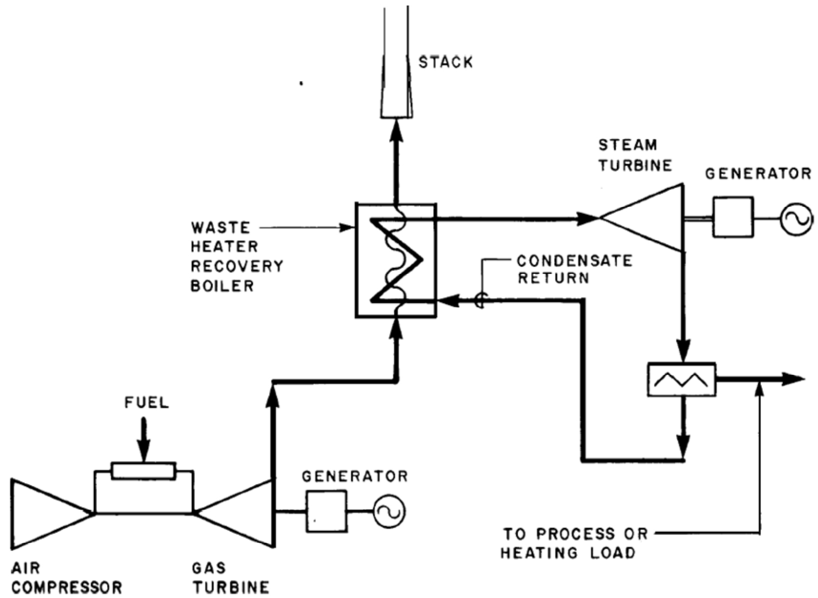

Figure 5: A Combined Cycle Unit with Single Gas Turbine and Single HRSG

However, EDC involving CC units is characterized as a non-convex optimization which is difficult, even impossible to solve by conventional methods. Many electric utilities prefer to represent their generator cost functions as single or multiple linear segment. Conventional EDC problems with piecewise linear cost functions can be solved by linear programming. Sheble [55] proposed a real-time economic dispatch algorithm - Merit Order Loading (MOL) based on the theory of linear programming which is very fast and efficient. Ongsakul [56] made a modification for MOL and sorted CC units based on the unit incremental cost at the highest outputs, but an example with only CC units was provided. Arroyo et al. [57] proposed to use piecewise linear approximation to solve EDC with nonconvex functions, the decision variables are generation levels of a block given the slope. The idea in [58] is to apply a stepwise function to approximate a nonlinear curve and transform a nonlinear programming to a mixed integer linear programming (MILP) formulation. Heuristic algorithms are also very popular to solve non-convex optimization.

Sheble et al. [59] proposed a refined genetic algorithm (RGA) method to solve EDC problems with non-convex cost curve considering valve point effects. Yang et al. [51] implemented an evolutionary programming for non-smoothing fuel cost functions. Wong [60] et al. proposed an evolutionary programming-based algorithm for EDC with environmentally constraints. Park et al. [61] presented a modified particle swarm optimization to solve EDC with multiple fuels.

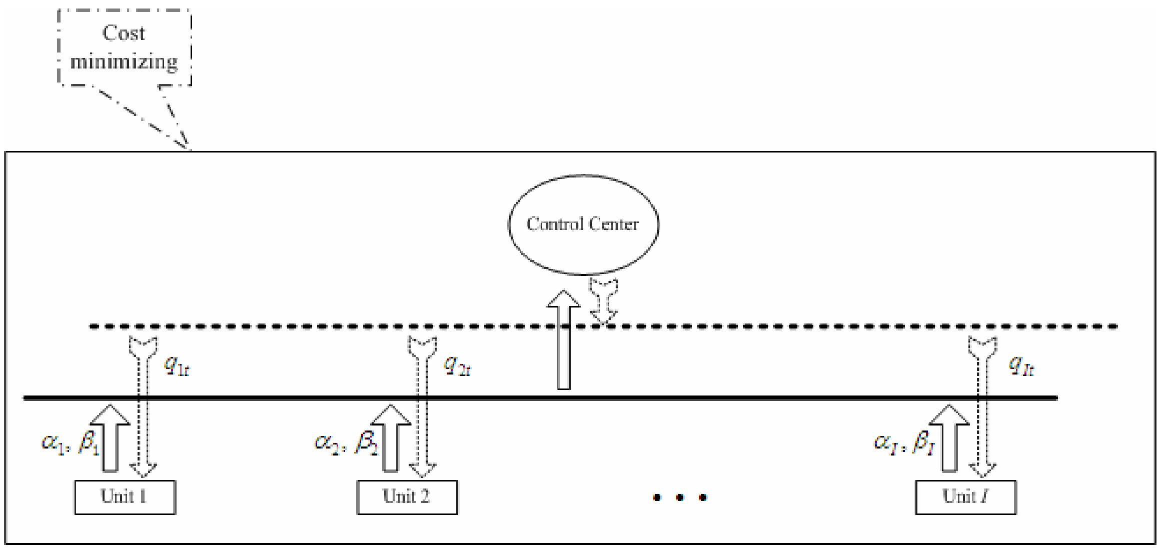

Figure 6: EDC within an Integrated Utility

This chapter first explains cost functions of combined cycle units; second proposes several novel algorithms including a Hybrid Technique (HT), two MILP formulations, and GA/EP/PS for EDC with combined cycle units; third, a mutation prediction technique is proposed to improve the efficiency of GA. Furthermore, the trajectory and searching path of each artificial life technique is shown and compared generation-by-generation in Chapter Five.

3.2 Combined Cycle Units Cost Curve

Typically, a combined cycle unit consists of several combustion turbines (CTs) and an HRSG/steam turbine (ST) set. Based on different combinations of CTs and STs, a combined cycle unit can operate at multiple configurations. Each combination of CTs and STs can be regarded as a state. Each state has its own unique cost characteristic. The heat rate of a modern CC unit varied from 9.0 to 11.1 GJ/MWH (8.5 to 10.5 MBTU/MWH). However the heat rate of a combustion turbine is about 15.8 GJ/MWH (15 MBTU/MWH) [62]. The EDC problem assumes that the state of a combined cycle unit is known aforehand and may be decided by unit commitment (UC) program.

Assuming a combined cycle unit consists of two combustion turbines and one HRSG/steam turbine [63]. All configurations are shown in Table 5:

Table 5: The States of a Combined Cycle Unit

State	Composition	Min Power	Max Power
1	1 CT	P_{1min}	P_{1max}
2	2 CT	P_{2min}	P_{2max}
3	1 CT + 1 ST	P_{3min}	P_{3max}
4	2 CT + 1 ST	P_{4min}	P_{4max}

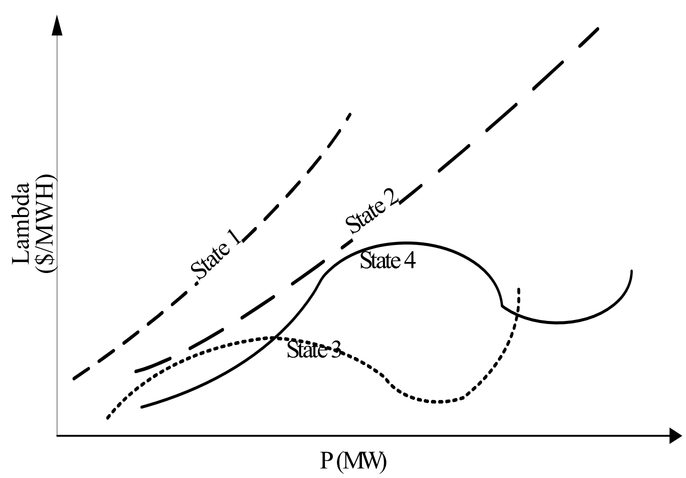

Figure 7: Incremental Cost Curves of a CC Unit

The incremental cost curves of state 3&4 are not monotonically increasing with generation and are depicted in Fig. 7. Lambda represents incremental cost ($/MWH), which is marginal cost of a unit, too.

Table 6: The States of a Combined Cycle Unit

State 1		**State 2**		**State 3**		**State 4**	
MW	$/H	MW	$/H	MW	$/H	MW	$/H
60	5026	120	10051	95	5026	190	10051
90	6084	180	12167	145	6084	290	12167
110	6771	220	13542	168	6771	335	13542
130	7602	260	15203	189	7602	378	15203
150	8469	300	16939	210	8469	420	16939
170	9390	340	18780	245	9390	490	18780
180	9903	360	19806	265	9903	530	19806
200	10876	400	21752	295	10876	590	21752

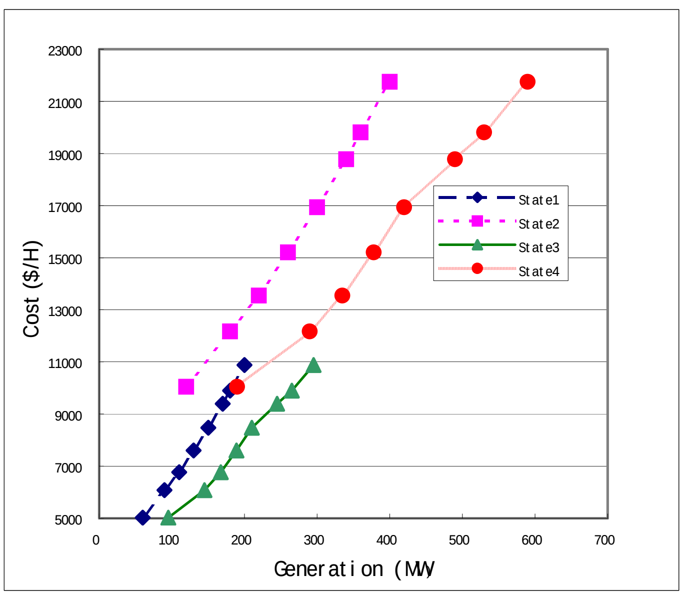

Figure 8: Piecewise Linear Cost Curves of a CC unit

Breakpoints of piece-wise linear cost curves of a CC unit are shown in Table 6. For the sake of simplicity, capital “H” represents “Hour”. Four piecewise linear cost curves of the CC unit are shown in Fig. 8. State 1&2 essentially are thermal unit states. Correspondingly, cost curves are monotonously increasing and convex. The curves labeled by “State1” and “State2” in Fig. 8 confirm the assertion. However, the curves labeled by “State3” and “State4” are monotonously increasing, but not convex any more for they are combined cycle unit states.

3.3 EDC Problem Formulation with CC Units

The classical EDC problem formulation is as follows:

Minimize: $F = \sum_{i=1}^{n} f_i(P_i)$ (15)

Subject to:
$$\sum_{i=1}^{n} P_i = P_D \Rightarrow g(P_i) = \sum_{i=1}^{n} P_i - P_D = 0$$
$$P_i \geq \underline{P_i}$$
$$P_i \leq \overline{P_i} \quad (16)$$

Where:

P_i: generation of unit i
f_i: cost of unit i
F: total cost of n units
P_D: total demand
$\underline{P_i}$: generation lower limit of unit i
$\overline{P_i}$: generation upper limit of unit i
n: the number of units

When each individual cost curves $fi(Pi)$, i=1,…,n, are convex, the objective function is also convex, because the summation of convex functions is also a convex function. Since all constraints are linear (Convex Programming), the Karush-Kuhn-Tucker condition guarantees that the problem has only one global minimum.

We form the Lagrange function as follows:

$$L = F(P_i) - \lambda(\sum_{i=1}^{n} P_i - P_D) \tag{17}$$

This fact allows us to find the solution by applying first-order necessary conditions, which are in this case:

$$\begin{aligned} &\frac{\partial L}{\partial P_i} = 0 \Rightarrow \frac{\partial f_i(P_i)}{\partial P_i} = \lambda, \quad i = 1,\ldots,n \\ &\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_{i=1}^{n} P_i - P_D = 0 \end{aligned} \tag{18}$$

Inequality constraints may be handled by checking whether the resulting solution is against them, and for any violation, setting up another equality constraint which binds the given decision variable to the limit which was violated.

If neither of the inequality constraints is binding, then $\frac{\partial f_i(P_i)}{\partial P_i} = \lambda$ induces the "equal incremental cost criterion" which is a simple and powerful rule to solve traditional ED problem.

Again, because F is convex and g is linear, we can assure a unique minimum solution will be found.

However, given combined cycle units' characteristics, the traditional EDC algorithms as shown before may fail to find the minimum solution. These points, which satisfy first–order necessary conditions, may be minimum points, or maximum points, or saddle points. For example: consider the break points of state 4 in Table 6 (column 7 and 8), a curve fitting is done to approximate the cost function by a 4th polynomial function. The polynomial function is shown by (19):

$$f(P) = 0.0000011850P^4 - 0.0020019P^3 + 1.2175P^2 - 282.07P + 31892 \tag{19}$$

Both breakpoints and fitting function are shown in Fig. 9.

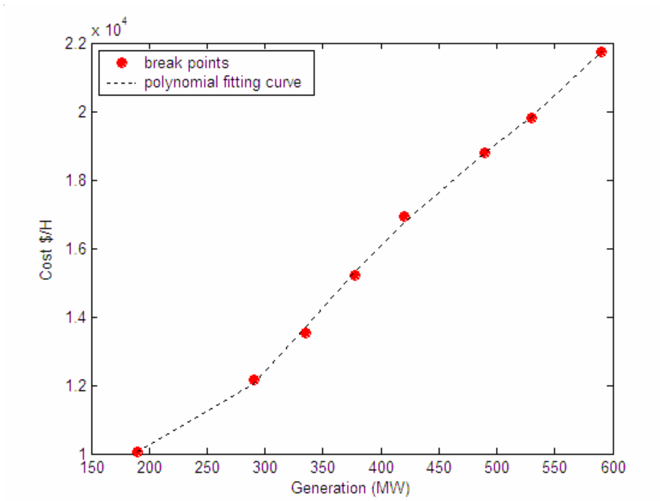

Figure 9: Breakpoints and Approximated Cost Function of a CC Unit

Assume that two identical CC units with the same approximated 4th order polynomial cost functions *f* are connected to a demand of 800 MW.

The EDC problem can be simply expressed as below:

Minimize: $F = f(P_1) + f(P_2)$ (20)

Subject to:

$$\begin{aligned} &P_1 + P_2 = 800 \\ &190 \le P_1 \le 590 \\ &190 \le P_2 \le 590 \end{aligned} \tag{21}$$

Where *f* is defined by (19).

If both cost functions of the two units are convex, the optimal solution should be symmetric, i.e. $P_1 = P_2 = 400$ MW. However, the symmetric solution is a local maximum in this case. Equal lambda criterion gives a wrong answer.

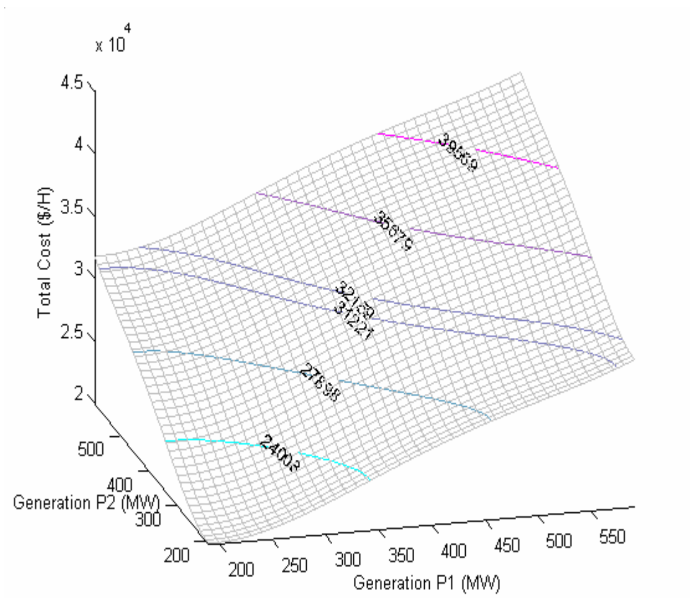

Figure 10: Six 3-D Contour Lines of Total Cost Function

Fig. 10 shows the objective function F with respect to P_1 and P_2 in a 3-dimension fashion. Six contour lines are sketched on the surface, they are 24008, 27898 31221, 32159, 35678, and 39568. Fig. 11 shows the projections of six contour lines on ground level. It is clearly seen that the total cost becomes larger and larger from southwest to northeast.

Finally, the equality constraint needs to be taken care. In a 3-D solution space, the equality constraint represents a plane, which is perpendicular to the ground level at the straight line $P_1 + P_2 = 800$. Fig. 12 shows the projection of the constraint on the ground level. It is observed that there are two contour lines tangent to the constraint. They are contour line 31221 and 32159. Since the objective function is going to increase from southwest to northeast, the symmetric tangent point (400, 400) should be a local maximum. However, there are two local minima: (259, 541) & (541, 259), which are marked in Fig. 12. According to the example, we find an interesting issue that even though two CC units are exactly identical, they may dispatch differently at the optimal solution. That is because of the non-convexity of cost functions.

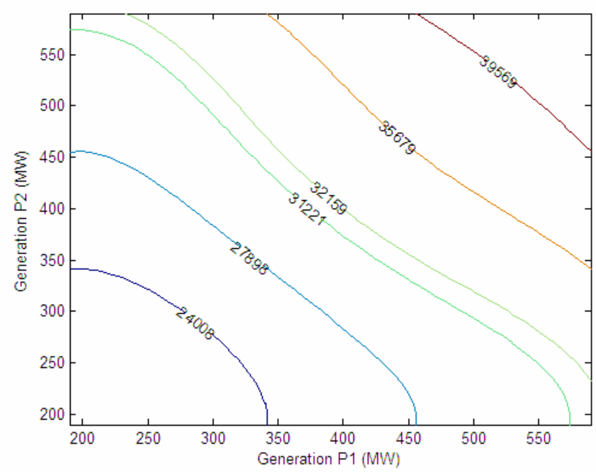

Figure 11: Six 2-D Contour Lines of Total Cost Function

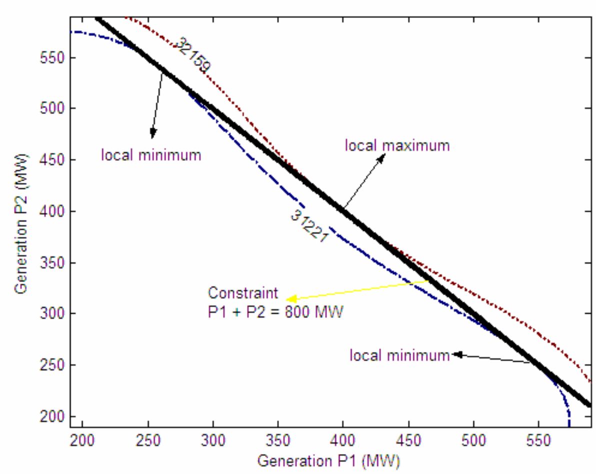

Figure 12: Local Minimum and Maximum of the Two-CC Units Example

3.4 Comparison of Solution Methods

3.4.1 Complete Enumeration

Complete enumeration means to search every possible solution within a feasible region to find the optimal one. The searching scheme is shown in Fig. 13. The basic idea is to discretize generation level of each unit, sequentially change generation output of one unit while keeping all of others constant until all of possible trials are examined. The algorithm will seek the minimal cost within all feasible solutions.

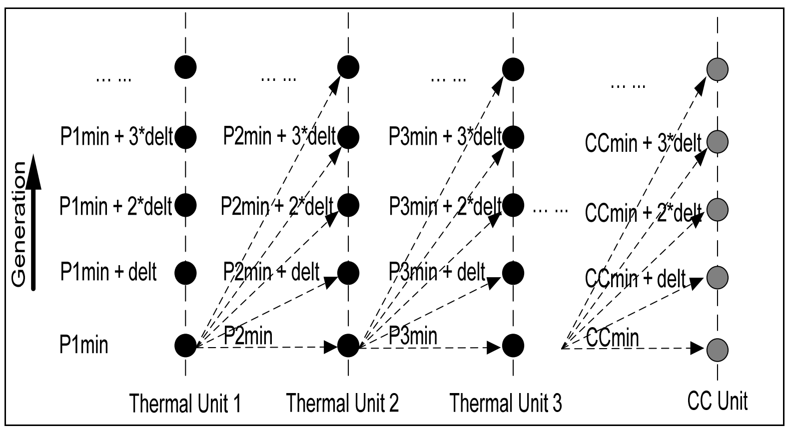

Figure 13: Searching Scheme of Complete Enumeration

Procedure:

Assume totally (*N*+*M*) units (*N* thermal units and *M* CC units) participate in economic dispatch. Set demand = *D*. *Delta* is the step size of the changing of generation;

Set i=1, j=0;

If i > N+M, go to step 7;

For unit *i*, set $P_i = P_{i,\,min} + j{*}Delta$, if $P_i > P_{i,\,max}$, go to step 6;

if $\sum_{i=1}^{N+M} P_i$ = D, store Pi and go to step 6; otherwise j = j + 1, go to step 4;

i = i + 1, go to step 3;

Within all of the solutions, select the lowest total cost and corresponding generation of each unit as the optimal solution.

3.4.2 Merit Order Loading

Merit order loading [55] is a fast algorithm for conventional EDC with thermal units. The process is to sort the unit-segments into ascending sequence by breakpoints of piece-wise linear incremental cost curves. The monotonously decreasing segments of incremental cost curves of combined cycle units could be handled. Reference [56] applied the original merit order loading method to CC units by ordering unit-segments at the highest/lowest outputs. Clearly, this approach is only an approximate method.

An illustration of the unit dispatching sequence of four combined cycle units has been shown in Fig. 14.

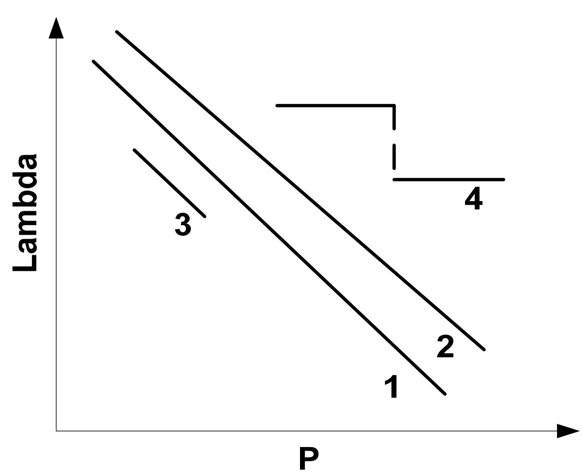

Figure 14: Dispatching Sequence of CC units by MOL

Procedure:

Assume totally N thermal units and M CC units participate in economic dispatch. Set demand = D;

Sort the unit-segments into ascending sequence by upper breakpoint of piece-wise linear ICs of thermal units;

Sort monotonous increasing sections of piece-wise linear ICs of CC units into ascending sequence by upper breakpoint;

Sort monotonous decreasing sections of piece-wise linear ICs of CC units into ascending sequence by maximal generation point (minimal incremental cost);

Dispatch the unit-segment by adding each incremental unit-segment generation into the total generation;

Increment the unit-segment index until all demand D is met.

3.4.3 Hybrid Technique

The hybrid technique originates from the idea that convex thermal units and CC units can be divided into two different groups, for conventional units group, convex optimization methods such as Lambda Iteration are applied; for CC units, Complete Enumeration is applied. This proposed technique combines Lambda Iteration with Complete Enumeration is shown in Table 7. Lambda Iteration is able to find the optimal cost for thermal units part as well as Complete Enumeration for CC units part. The global optimum can be reached whatever the order of the CC units considered within the algorithm is. Essentially HT is to build a composite generation cost function for all of thermal units, then apply CE to solve a lower dimension non-convex optimization.

Table 7: Illustration of Hybrid Technique

Conventional Units		**CC Units**		**Total**
Lambda Iteration		Complete Enumeration		Solution
Generation Sum	+	Generation Sum	=	Generation
Cost Sum	+	Cost Sum	=	Cost
Demand served	+	Demand served	=	Demand

Procedure:

Assume totally N thermal units and M CC units participate in economic dispatch. Set demand = D, $i = 0$ and P_0 is the summation of generation lower limit of all M CC units, ΔP is step size of the changing of generation;

Calculate the cost of all of CC units with demand $P = P_0 + i * \Delta P$ by Complete Enumeration. If $P >$ the summation of generation upper limit of all M CC units, go to step 6;

Using Lambda Iteration to dispatch generation ($D - P$), and calculate the cost of each thermal unit;

Calculate the total cost of CC units and thermal units;

$i = i + 1$, go to step 2;

Find the minimal total cost among all of the solutions, it is the optimal solution.

3.4.4 Mixed Integer Linear Programming Model

3.4.4.1 Linear Programming Formulation

Assuming there are two thermal units and one CC unit to participate in economic dispatch, each unit is represented by a two-segment piece-wise linear cost curve. Thermal unit has a convex cost curve while CC unit has a concave cost curve. Cost curves of these three units are shown in Figure 15, 16 and 17:

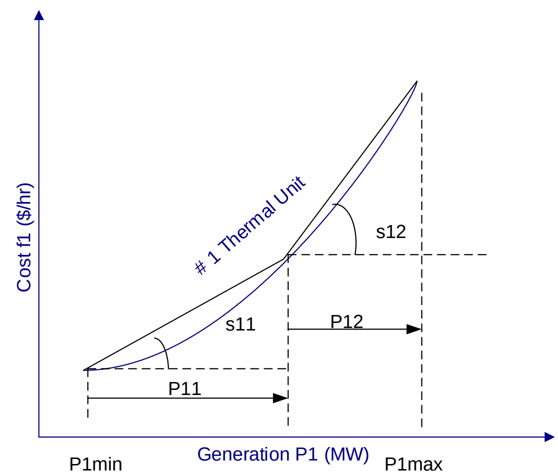

Figure 15: Cost Curves of # 1 Thermal Unit

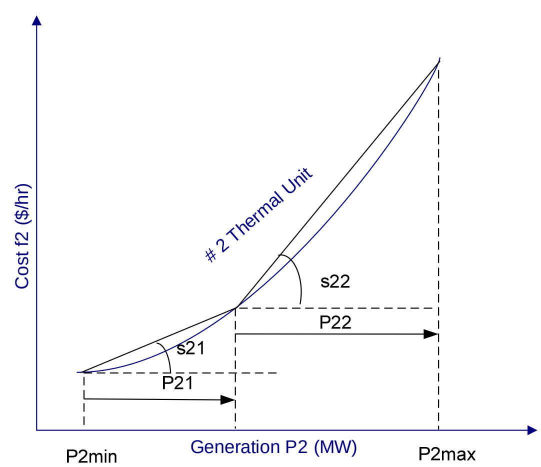

Figure 16: Cost Curve of # 2 Thermal Unit

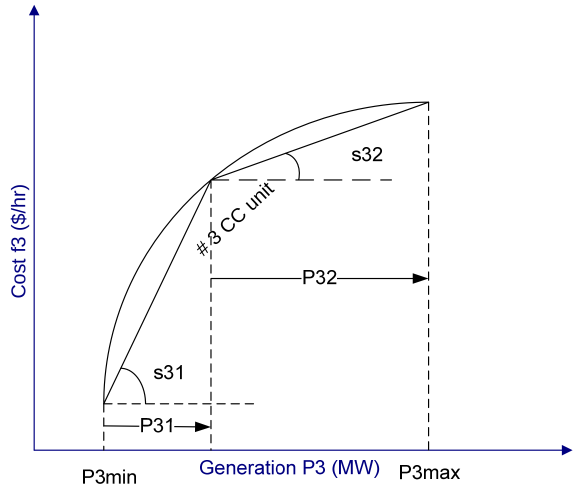

Figure 17: Cost Curve of # 3 CC Unit

A nonlinear function can be approximated with a series of piecewise linear segments. In Fig. 15, the variables P_{11} and P_{12} represent generation increments that range from 0 to some maximum values $P_{11,max}$ and $P_{12,max}$, respectively.

Thus, we have that

$$0 < P_{11} < P_{11,\max}$$
$$0 < P_{12} < P_{12,\max}$$

And

$$P_1 = P_{1\min} + P_{11} + P_{12}$$

Let's denote the slope of each one of the line segments as s_{11} and s_{12}, respectively. Then the increment in cost function f_1 corresponding to each line segment is given by

$$\Delta\hat{f}_{11} = s_{11}P_{11}$$
$$\Delta\hat{f}_{12} = s_{12}P_{12}$$

So the cost function can be approximated using the line segments, and that approximation may be improved to any desired level by increasing the number of line segments used.

Let's denote the approximate cost function as $\hat{f}_1$, which can be expressed as a function of the new variables P_{ij}, according to:

$$\hat{f}_1(P_{11}, P_{12}) = f_1(P_{1min}) + s_{11}P_{11} + s_{12}P_{12}$$

By the same way, cost curves of unit # 2 and # 3 can be represented by piecewise linear functions.

$$\hat{f}_2(P_{21}, P_{22}) = f_2(P_{2min}) + s_{21}P_{21} + s_{22}P_{22}$$

$$0 < P_{21} < P_{21,\max}$$
$$0 < P_{22} < P_{22,\max}$$
$$P_2 = P_{2\min} + P_{21} + P_{22}$$

And

$$\hat{f}_3(P_{31}, P_{32}) = f_3(P_{3min}) + s_{31}P_{31} + s_{32}P_{32}$$

$$0 < P_{31} < P_{31,\max}$$
$$0 < P_{32} < P_{32,\max}$$
$$P_3 = P_{3\min} + P_{31} + P_{32}$$

An important observation is, to thermal unit # 1 & # 2, $s_{11} < s_{12}$ and $s_{21} < s_{22}$ because of convexity of cost curves. The property guarantees to give the highest priority to using P_{i1} when increasing P_i from P_{imin}, the next highest priority to using P_{i2}, and so on if minimizing $\hat{f}_i$.

However, to CC unit # 3, it is not be guaranteed to give the highest priority to using P_{31}, the next highest priority to using P_{32}, and so on when minimizing $\hat{f}_3$, because of concavity of cost curves. Therefore a special constraint needs to be added: [64]

$$P_{31} < P_{31,max} \Rightarrow P_{32} = 0$$

This equation can be re-written as below:

$(P_{31} - P_{31,max}) * P_{32} = 0$

Which assures $P_{32} = 0$ whenever $P_{31} < P_{31,max}$

Therefore, the optimization problem can be re-written as follow:

min $\hat{F} = s_{11}P_{11}+s_{12}P_{12}+ s_{21}P_{21}+s_{22}P_{22}+s_{31}P_{31}+s_{32}P_{32}$

s.t.

$$0 < P_{11} < P_{11,\max}$$
$$0 < P_{12} < P_{12,\max}$$
$$0 < P_{21} < P_{21,\max}$$
$$0 < P_{22} < P_{22,\max}$$
$$0 < P_{31} < P_{31,\max}$$
$$0 < P_{32} < P_{32,\max}$$
$$P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} = P_D - (P_{1\min} + P_{2\min} + P_{3\min})$$
$$(P_{31} - P_{31,\max}) * P_{32} = 0$$

The minimal cost should be equal to

$$\hat{f}_1(P_{1min}) + \hat{f}_2(P_{2min}) + \hat{f}_3(P_{3min}) + s_{11}P_{11} + s_{12}P_{12} + s_{21}P_{21} + s_{22}P_{22} + s_{31}P_{31} + s_{32}P_{32}$$

The outputs of units should be

$$P_1 = P_{1\min} + P_{11} + P_{12}$$
$$P_2 = P_{2\min} + P_{21} + P_{22}$$
$$P_3 = P_{3\min} + P_{31} + P_{32}$$

3.4.4.2 Mixed Integer Linear Programming Formulation (I)

In order to drop the special constraint, one has to bring in some integer variables. To CC unit # 3, it is not be guaranteed to give the highest priority to using P_{31}, the next highest priority to using P_{32}, and so on when minimizing $\hat{f}_3$, because of concavity of cost curves. A special constraint needs to be added:

$$P_{31} < P_{31,max} \Rightarrow P_{32} = 0$$

We can bring in an integer variable to reformulate the special constraint. Denote by M as a large enough positive number, y_1 is a binary variable with a value 0 or 1.

The special constraint can be changed into two equations: [65]

$$P_{32} \leq My_1$$
$$P_{31} - P_{31,\max} \geq M(y_1 - 1)$$

It is clear that if $P_{31} < P_{31,max}$ then $y_1 = 0$, $P_{32} \leqslant 0$; if $P_{31} \geq P_{31,max}$ then y_1 and P_{32} are unrestricted.

With the two new constraints, the optimization problem can be re-written as follow:

$$\min \ \hat{F} = s_{11}P_{11}+s_{12}P_{12}+ s_{21}P_{21}+s_{22}P_{22}+s_{31}P_{31}+s_{32}P_{32}$$

s.t.

$$0 < P_{11} < P_{11,\max}$$
$$0 < P_{12} < P_{12,\max}$$
$$0 < P_{21} < P_{21,\max}$$
$$0 < P_{22} < P_{22,\max}$$
$$0 < P_{31} < P_{31,\max}$$
$$0 < P_{32} < P_{32,\max}$$
$$P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} = P_D - (P_{1\min} + P_{2\min} + P_{3\min})$$
$$P_{32} \leq My_1$$
$$P_{31} - P_{31,\max} \geq M(y_1 - 1)$$

M is a large positive number

$y_1 = 0$ or 1

This is a Mixed Integer Linear Programming model, which can be solved by Branch-and-Bound algorithm.

3.4.4.3 Mixed Integer Linear Programming Formulation (II)

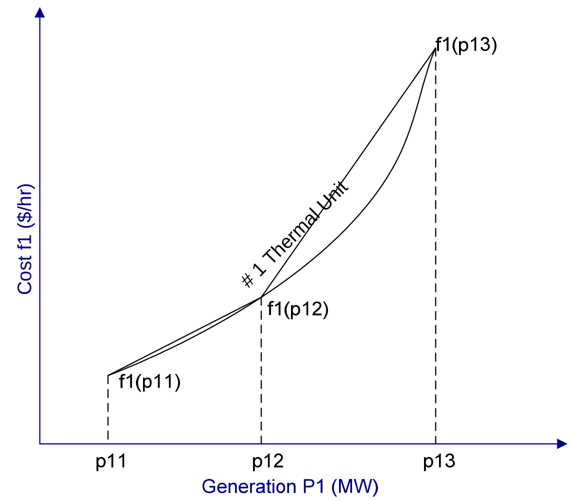

Figure 18: Breakpoints of # 1 Thermal Unit

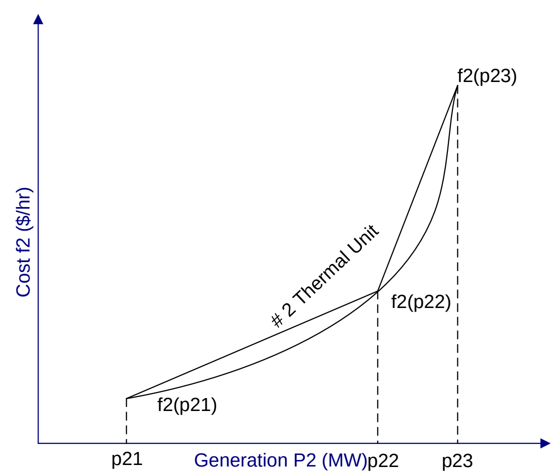

Figure 19: Breakpoints of # 2 Thermal Unit

In section 3.4.4.2, it is necessary to assign a large positive number M for the algorithm. Sometimes, the improper M value will affect the precision of the algorithm. So an alternative mixed integer linear programming model is proposed without assigning M value [66].

Figure 18, 19, 20 show all of breakpoints on cost curves of unit 1, 2, 3.

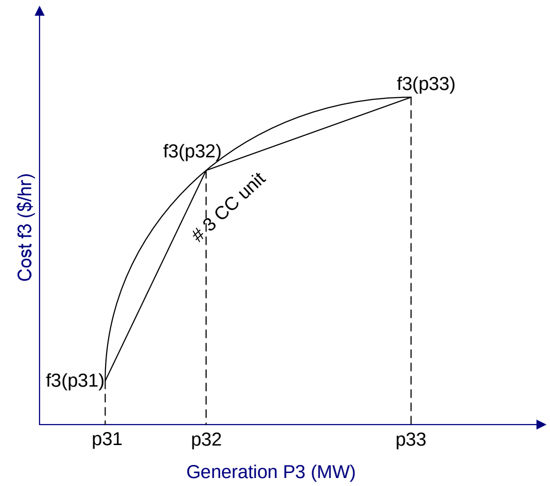

Figure 20: Breakpoints of # 3 CC Unit

Denote all of breakpoints of cost curves as follows

$$f_1(p_{11}) = f_{11}$$
$$f_1(p_{12}) = f_{12}$$
$$f_1(p_{13}) = f_{13}$$
$$f_2(p_{21}) = f_{21}$$
$$f_2(p_{22}) = f_{22}$$
$$f_2(p_{23}) = f_{23}$$
$$f_3(p_{31}) = f_{31}$$
$$f_3(p_{32}) = f_{32}$$
$$f_3(p_{33}) = f_{33}$$

Define 3 real variables x_{11}, x_{12}, and x_{13} within the interval [0,1], the cost function of thermal unit # 1 can be expressed as below:

$$\tilde{f}_1 = x_{11} f_{11} + x_{12} f_{12} + x_{13} f_{13}$$

$$x_{11} + x_{12} + x_{13} = 1$$
$$x_{11} \sim x_{13} \geq 0$$

Similarly, define 3 real variables x_{21}, x_{22}, and x_{23} within the interval [0,1], the cost function of thermal unit # 2 is expressed as below:

$$\tilde{f}_2 = x_{21} f_{21} + x_{22} f_{22} + x_{23} f_{23}$$

$$x_{21} + x_{22} + x_{23} = 1$$
$$x_{21} \sim x_{23} \geq 0$$

Then define 3 real variables x_{31}, x_{32}, and x_{33} within the interval [0,1]; 2 binary variables y_{31}, y_{32}, the cost function of CC unit # 3 can be expressed as below:

$$\tilde{f}_3 = x_{31} f_{31} + x_{32} f_{32} + x_{33} f_{33}$$

$$x_{31} - y_{31} \leq 0$$
$$x_{32} - y_{31} - y_{32} \leq 0$$
$$x_{33} - y_{32} \leq 0$$
$$y_{31} + y_{32} + y_{33} = 1$$
$$x_{31} + x_{32} + x_{33} = 1$$
$$x_{31} \sim x_{33} \geq 0$$
$$y_{31}, y_{32} = 0 or 1$$

With the notations above, the optimization problem can be re-written as follow:

$$
\begin{aligned}
& min\tilde{F} = x_{11}f_{11} + x_{12}f_{12} + x_{13}f_{13} + x_{21}f_{21} + x_{22}f_{22} + x_{23}f_{23} + x_{31}f_{31} + x_{32}f_{32} + x_{33}f_{33} \\
& s.t. \\
& x_{11} + x_{12} + x_{13} = 1 \\
& x_{21} + x_{22} + x_{23} = 1 \\
& x_{31} + x_{32} + x_{33} = 1 \\
& x_{31} - y_{31} \leq 0 \\
& x_{32} - y_{31} - y_{32} \leq 0 \\
& x_{33} - y_{32} \leq 0 \\
& y_{31} + y_{32} + y_{33} = 1 \\
& x_{11}p_{11} + x_{12}p_{12} + x_{13}p_{13} + x_{21}p_{21} + x_{22}p_{22} + x_{23}p_{23} + x_{31}p_{31} + x_{32}p_{32} + x_{33}p_{33} = P_D \\
& x_{ij} \geq 0 \\
& y_{31}, y_{32} = 0 or 1 \\
& i = 1,2,3 \\
& j = 1,2,3
\end{aligned}
$$

Since the generation level for each unit is represented as a combination of all its breakpoints, the maximal and minimal generation limit are automatically satisfied. Therefore, the maximal and minimal generation limit constraints can be deleted. This model can be solved by Branch-and-Bound algorithm.

3.4.5 Genetic Algorithm – Mutation Prediction

Mutation is an important operator. In SGA, the random number generator needs to be called each time for each bit to decide whether to carry out mutation operation or not. By reducing the number of times that the random number generator is called, computing time can be minimized. This is the motivation of mutation prediction.

In order to understand the mechanism of mutation prediction, two strategies need be compared. Assume there are totally m bits in one-generation chromosomes.

First strategy: for each bit, toss a coin. If head, mutate; otherwise do not mutate. Assume the probability of head is Q, the probability of tail should be 1-Q, i.e. Q represents the probability of mutation of each bit.

Define a random variable X which is the number of heads (mutations) in m trials. We know X satisfies Binomial Distribution.

$$\begin{aligned}&\text{Prob}(X=k)=C_m^k Q^k (1-Q)^{m-k}\\&k=0,1,\ldots m\end{aligned} \tag{22}$$

The expected value of X is

$$E(X)=mQ \tag{23}$$

Which means the average number of mutations in m bits is mQ.

Second strategy: for the same case of tossing a coin above, define another random variable Y which is the number of trials when head (mutation) first appears. We know Y satisfies Geometric Distribution.

$$\begin{aligned}&\text{Prob}(Y=h)=(1-Q)^{h-1}Q\\&h=1,2,\ldots,m\end{aligned} \tag{24}$$

The expected value of Y is

$$E(Y)=1/Q \tag{25}$$

Which means the average number of trials when head first appears is $1/Q$.

Therefore within m bits, the average number of heads (mutations) is

$$\frac{m}{E(Y)}=\frac{m}{1/Q}=mQ \tag{26}$$

It is easy to find equation (23) and (26) give the same result. However, the second strategy calls random number generator only one time, whereas, the first one calls random number generator one time per bit.

Therefore, mutation prediction should be carried out as follows: with $1/Q$ as the expectation parameter, randomly generate a series of numbers satisfying Geometric Distribution. These random numbers indicate positions of bits which need to be mutated.

CHAPTER 4. SUPPLY FUNCTION EQUILIBRIUM

4.1 Market Structure

GENCOs' optimal bidding strategies are related to actual market structure and market rule. Although there are other market mechanism designs, a power pool with uniform non-discriminatory pricing is popular and has been used in many electricity markets around the world. In this chapter, we only consider this pool-type energy market.

The market rules are assumed as follow: Every morning GENCOs are required to submit a series of bidding functions for the following T periods of the next day. T can be 24 or 48 etc, which indicates a day-head market trading is considered. After a market clearing mechanism, maximizing social welfare, is run by ISO, each GENCO is informed of the market price and awarded MW quantity for every period t. Then GENCOs can settle with ISO and calculate their profit/loss for the next day. A particular point should be mentioned is that if transmission congestion is not considered (Section 2.3), the market clearing price is the same for all players. In this case, market clearing condition is equivalent to maximizing social welfare. An integrated buyer is assumed with a linear demand function for each time t. However, if transmission congestion is incorporated (Section 4.3), the prices are usually different to players who are located at different nodes. A set of linear demand functions are assumed for loads at different location. In both of cases, the market rule is implemented by a single side auction *i.e.* GENCO's side auction only.

4.2 SFE with Multiple Periods

Notation:

t: index of time periods; $t = 1, 2, \ldots, T$; T: the number of time intervals;

i: index of GENCOs; $i = 1, 2, \ldots, I$; I: the number of GENCOs;

P_t: market clearing price at t;

Q_t: integrated demand at t;

q_{it}: generation of GENCO i at t;

l_{it}: GENCO i's decision variable (the intercept of supply function, *i.e.* l_{it} – parameterization) at t;

Market clearing condition at t: $Q_t = \sum_{i=1}^{I} q_{it}$;

Integrated demand curve at t: $P_t = h_t - g_t Q_t$ ($g_t > 0$);

GENCO i's bidding function (supply function) at t: $P_t = l_{it} + k_i q_{it}$ ($k_i > 0$);

GENCO i's cost function at t: $Cost_i(q_{it}) = \alpha_i q_{it} + 0.5\beta_i q_{it}^2$, $(\beta_i > 0)$;

GENCO i's profit function at t: $L_i^t = \pi_{it} = P_t q_{it} - Cost_i(q_{it}) = P_t q_{it} - \alpha_i q_{it} - 0.5\beta_i q_{it}^2$;

GENCO i's internal state variable (such as inventory level of fuel) at t: X_{it}; X_{i1} is given $\forall i \in I$;

GENCO i's salvage value of fuel at the end of the time horizon T: $\phi_i = \gamma_i X_{iT}$, X_{iT} is unknown, *i.e.* "free final state"; $\gamma_i > 0$ known; γ_i can be assumed to be the future/forward price of fuel (for thermal units);

GENCO i's heat rate: $H_i(q_{it}) = \delta_i Cost_i(q_{it}) = \delta_i(\alpha_i q_{it} + 0.5\beta_i q_{it}^2)$; δ_i can be assumed to be the reciprocal of fuel cost;

GENCO i's internal state dynamics: $f_i^t = X_{it+1} = X_{it} - \delta_i(\alpha_i q_{it} + 0.5\beta_i q_{it}^2)$;

As discussed in section 2.3, l_{it} – parameterization is more proper than k_{it} – parameterization for a SFE model with multiple periods. Therefore, l_{it} – parameterization will be analyzed in the rest of the study.

In a multiple period model, GENCO *i* faces a multi-stage decision making problem. The objective function switches to maximize its total profit over a certain time interval *T*, not at a time point *t*, plus the salvage value of physical resources left at the end of *T*. Another distinct change from a single period model is that GENCO *i* has to satisfy its inter-temporal internal physical resource constraint, which could be reservoir level of a hydro unit or fuel inventory of a thermal unit or a physical contract (Take-or-Pay fuel contract) etc. A multi-stage decision making problem of GENCO *i* is shown in Fig. 21.

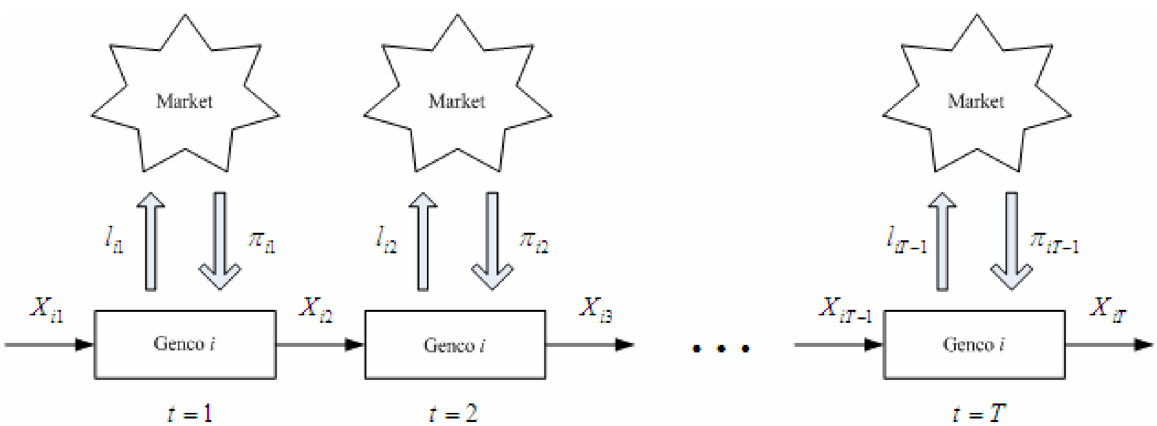

Figure 21: GENCO *i* Multi-stage Decision Making Problem

Therefore, the basic problem of SFE with multiple periods is that given a certain amount of physical resource; find a series of GENCO *i*'s optimal bidding functions over a time of periods *T*, which maximize GENCO *i*'s inter-temporal profit.

From GENCO *i*'s point of view, Hobbs etc. defines a "mathematical program with equilibrium constraints" (MPEC), which is a two-level constrained optimization problem [24]. The second level problem is that ISO maximizes the social welfare by determining the quantity and price of each GENCO given their “virtual” cost functions/supply functions bidding. The first level problem is that GENCO *i* maximizes its profit by choosing the parameters of supply function given the quantity and price determined in the second level problem. A penalty interior point algorithm (PIPA) is proposed to solve the single GENCO’s problem.

In a game theoretic context, the multi-GENCO problem can be phrased as a SFE, each being a dominant GENCO with respect to the ISO, able to predict how the ISO will process the bids of all players (Stackberg Game). The main feature of the game is that each GENCO is solving a MPEC, rather than a standard optimization problem. In this multi-GENCO case, each GENCO is trying to maximize its profits based on what the market does as well as what the other dominant GENCOs do. A SFE exists when there is no incentive for any GENCO to change its behavior unilaterally [24]. Fig. 22 shows the two-level optimization problem.

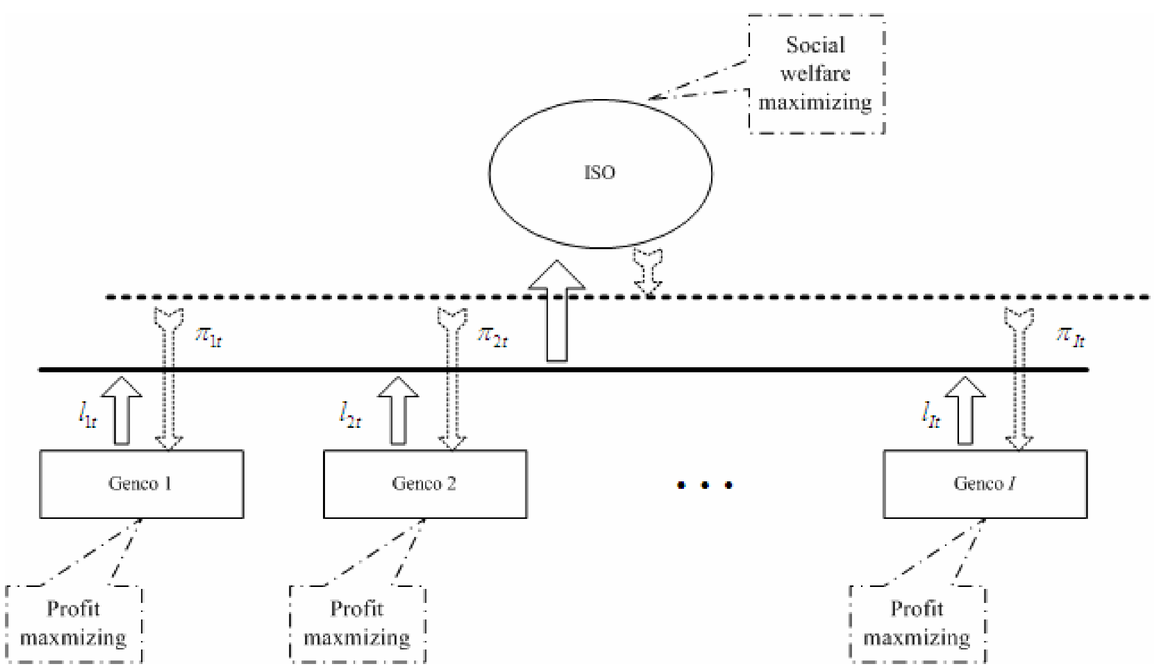

Figure 22: 2 – level optimization problem at time *t*

Since a multiple period problem is studied in this chapter, a GENCO's internal state constraint should be incorporated into its profit function. For GENCO *i*, its multiple periods problem is:

$$\begin{aligned} \max_{l_{it}} \quad & J_i = \phi_i(X_{iT}) + \sum_{t=1}^{T-1} L_i^t(l_{it}, l_{-it}) \\ s.t. \quad & X_{it+1} = X_{it} - \delta_i(\alpha_i q_{it} + 0.5\beta_i q_{it}^2) \\ & X_{i1} \text{ is given} \end{aligned}$$

From the perspective of GENCO *i,* it faces a free final state discrete time optimal control problem if other GENCOs' biddings are known. However, normally GENCO *i,* does not know other GENCOs' bidding. GENCO *i*'s profit clearly depends on other GENCOs' bidding. A SFE can be defined below.

A strategies profile $\left\{l_{it}^*, \text{ for all } i = 1, ..., I,\ t = 1,..,T\right\}$ is called to constitute a Supply Function Nash Equilibrium, if, with this bids l_{it}^* , GENCO *i* maximizes his payoff $J_i\left(X_{iT}, l_{it}, l_{-it}\right)$, given all of other GENCOs stick to the bid l_{-it}^* . The reasoning must hold for $\forall i \in I$.

It looks like that one must solve totally *I* discrete time optimal control problems simultaneously. However, some nice structures will decouple this problem and make it analytically solvable.

First of all, the Hamiltonian function is defined for GENCO *i*,

$$H_i^t = L_i^t(l_{it}, l_{-it}) + \lambda_{it+1}^T f_i^t(X_{it}, l_{it}, l_{-it})$$

where: λ_{it} is Lagrangian multipliers or costate variables

Assuming that the Nash Equilibrium strategy profile does exist. Then first-order condition for the discrete time optimal control problem must hold for every player simultaneously. Player *i*'s FOC is given below [67]:

(State Dynamics) $X_{it+1} = \dfrac{\partial H_i^t}{\partial \lambda_{it+1}} = f_i^t = X_{it} - \delta_i(\alpha_i q_{it} + 0.5\beta_i q_{it}^2)$

(Costate Dynamics) $\lambda_{it} = \dfrac{\partial H_i^t}{\partial X_{it}} = \dfrac{\partial L_i^t}{\partial X_{it}} + \lambda_{it+1}\dfrac{\partial f_i^t}{\partial X_{it}}$

$\because \dfrac{\partial L_i^t}{\partial X_{it}} = 0, \ \dfrac{\partial f_i^t}{\partial X_{it}} = 1$

$\therefore \lambda_{it} = 0 + \lambda_{it+1} 1 = \lambda_{it+1}$

(Stationary Condition) $0 = \dfrac{\partial L_i^t}{\partial l_{it}} + \left[\dfrac{\partial f_i^t}{\partial l_{it}}\right]^T \lambda_{it+1}$

(Boundary Condition) $\lambda_{iT} = \dfrac{\partial \phi_i}{\partial X_{it}} = \gamma_i$

where: $\begin{cases} f_i^t = X_{it} - \delta_i(\alpha_i q_{it} + 0.5\beta_i q_{it}^2) \\ L_i^t = P_i q_{it} - (\alpha_i q_{it} + 0.5\beta_i q_{it}^2) \end{cases}$

If combining costate dynamics and boundary condition, one immediately finds that the costate variables are constant, *i.e.* $\lambda_{it} = \lambda_{iT} = \gamma_i \quad \forall t = 1, \ldots, T$. Therefore the stationary condition is decoupled with respect to time *t*. We can actually solve the problem analytically.

The market clearing condition is given below:

$$\sum_{i=1}^{I} q_{it} = Q_t \Rightarrow \sum_{i=1}^{I} \frac{P_t - l_{it}}{k_i} = \frac{h_t - P_t}{g_t}$$

One can obtain the market clearing price and the quantity of GENCO *i*:

$$P_t = \frac{\sum_{i=1}^{I} \frac{l_{it}}{k_i} + \frac{h_t}{g_t}}{\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t}}, \quad q_{it} = \frac{P_t - l_{it}}{k_i} = \frac{\frac{\sum_{i=1}^{I} \frac{l_{it}}{k_i} + \frac{h_t}{g_t}}{\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t}} - l_{it}}{k_i}$$

Furthermore, one has the first-order derivative of the market clearing price and quantity of GENCO *i* with respect to GENCO *i*'s decision variable:

$$\dot{q}_{it} = \frac{\partial q_{it}}{\partial l_{it}} = \frac{\frac{\frac{1}{k_i}}{\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t}} - 1}{k_i} = \frac{1}{k_i^2 \left(\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t} \right)} - \frac{1}{k_i}, \quad \dot{P}_t = \frac{\partial P_t}{\partial l_{it}} = \frac{\frac{1}{k_i}}{\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t}} = \frac{1}{k_i \left(\sum_{i=1}^{I} \frac{1}{k_i} + \frac{1}{g_t} \right)}$$

Therefore,

$$\frac{\partial f_i^t}{\partial l_{it}} = -\delta_i \alpha_i \frac{\partial q_{it}}{\partial l_{it}} - \delta_i \beta_i q_{it} \frac{\partial q_{it}}{\partial l_{it}} = -\delta_i \alpha_i \dot{q}_{it} - \delta_i \beta_i q_{it} \dot{q}_{it}$$

$$\frac{\partial L_i^t}{\partial l_{it}} = P_t \frac{\partial q_{it}}{\partial l_{it}} + \frac{\partial P_t}{\partial l_{it}} q_{it} - \alpha_i \frac{\partial q_{it}}{\partial l_{it}} - \beta_i q_{it} \frac{\partial q_{it}}{\partial l_{it}} = P_t \dot{q}_{it} + \dot{P}_t q_{it} - \alpha_i \dot{q}_{it} - \beta_i q_{it} \dot{q}_{it}$$

According to the stationary condition:

$$0 = \frac{\partial L_i^t}{\partial l_{it}} + \left[\frac{\partial f_i^t}{\partial l_{it}} \right]^T \lambda_{it+1} \quad i.e. \quad \frac{\partial L_i^t}{\partial l_{it}} = -\gamma_i \frac{\partial f_i^t}{\partial l_{it}}$$

One can obtain:

$$\alpha_i\delta_i\gamma_i\dot{q}_{it} + \beta_i\delta_i\gamma_i q_{it}\dot{q}_{it} = P_t\dot{q}_{it} + \dot{P}_t q_{it} - \alpha_i\dot{q}_{it} - \beta_i q_{it}\dot{q}_{it} \Rightarrow$$
$$\alpha_i\delta_i\gamma_i\dot{q}_{it} + \alpha_i\dot{q}_{it} = P_t\dot{q}_{it} + \dot{P}_t q_{it} - \beta_i\delta_i\gamma_i q_{it}\dot{q}_{it} - \beta_i q_{it}\dot{q}_{it} \Rightarrow$$
$$\left(1+\delta_i\gamma_i\right)\alpha_i\dot{q}_{it} = \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)q_{it} + P_t\dot{q}_{it}$$

Substituting GENCO *i*' supply function into equation:

$$\left(1+\delta_i\gamma_i\right)\alpha_i\dot{q}_{it} = \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)\frac{P_t - l_{it}}{k_i} + P_t\dot{q}_{it}$$

Doing some rearrangement:

$$\left(1+\delta_i\gamma_i\right)\alpha_i k_i\dot{q}_{it} = \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)\left(P_t - l_{it}\right) + k_i\dot{q}_{it}P_t$$
$$= \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)P_t - \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)l_{it}$$
$$= \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)\frac{\sum_{i=1}^{I}\frac{l_{it}}{k_i} + \frac{h_t}{g_t}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}} - \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)l_{it}$$
$$= \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)\frac{\sum_{i=1}^{I}\frac{l_{it}}{k_i}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}} + \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)\frac{\frac{h_t}{g_t}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}$$
$$-\left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)l_{it}$$

Segregating these terms with GENCOs' decision variables l_{it} from others terms:

$$\left(1+\delta_i\gamma_i\right)\alpha_i k_i\dot{q}_{it} - \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)\frac{\frac{h_t}{g_t}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}$$
$$= \left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it} + k_i\dot{q}_{it}\right)\left(\frac{\frac{1}{k_1}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}l_{1t} + \frac{\frac{1}{k_2}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}l_{2t} + \cdots + \frac{\frac{1}{k_i}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}l_{it} + \cdots + \frac{\frac{1}{k_I}}{\sum_{i=1}^{I}\frac{1}{k_i} + \frac{1}{g_t}}l_{It}\right)$$
$$-\left(\dot{P}_t - \beta_i\delta_i\gamma_i\dot{q}_{it} - \beta_i\dot{q}_{it}\right)l_{it}$$

Define three variables that are not dependent on GENCOs' decision variables as follows:

$$A_{it}=(1+\delta_i\gamma_i)\alpha_i k_i\dot{q}_{it}-\left(\dot{P}_t-\beta_i\delta_i\gamma_i\dot{q}_{it}-\beta_i\dot{q}_{it}+k_i\dot{q}_{it}\right)\frac{\frac{h_t}{g_t}}{\sum_{i=1}^{I}\frac{1}{k_i}+\frac{1}{g_t}}=(1+\delta_i\gamma_i)\alpha_i k_i\dot{q}_{it}-B_{it}\frac{h_t}{g_t},$$

$$B_{it}=\left(\dot{P}_t-\beta_i\delta_i\gamma_i\dot{q}_{it}-\beta_i\dot{q}_{it}+k_i\dot{q}_{it}\right)\frac{1}{\sum_{i=1}^{I}\frac{1}{k_i}+\frac{1}{g_t}}=(C_{it}+k_i\dot{q}_{it})\frac{1}{\sum_{i=1}^{I}\frac{1}{k_i}+\frac{1}{g_t}},$$

$$C_{it}=\dot{P}_t-\beta_i\delta_i\gamma_i\dot{q}_{it}-\beta_i\dot{q}_{it}$$

Then one has:

$$\begin{aligned}
A_{1t}&=\left(B_{1t}\frac{1}{k_1}-C_{1t}\right)l_{1t}+B_{1t}\frac{1}{k_2}l_{2t}+\cdots+B_{1t}\frac{1}{k_i}l_{it}+\cdots+B_{1t}\frac{1}{k_I}l_{It}\\
A_{2t}&=B_{2t}\frac{1}{k_1}l_{1t}+\left(B_{2t}\frac{1}{k_2}-C_{2t}\right)l_{2t}+\cdots+B_{2t}\frac{1}{k_i}l_{it}+\cdots+B_{2t}\frac{1}{k_I}l_{It}\\
&\vdots\\
A_{it}&=B_{it}\frac{1}{k_1}l_{1t}+B_{it}\frac{1}{k_2}l_{2t}+\cdots+\left(B_{it}\frac{1}{k_i}-C_{it}\right)l_{it}+\cdots+B_{it}\frac{1}{k_I}l_{It}\\
&\vdots\\
A_{It}&=B_{It}\frac{1}{k_1}l_{1t}+B_{It}\frac{1}{k_2}l_{2t}+\cdots+B_{It}\frac{1}{k_i}l_{it}+\cdots+\left(B_{It}\frac{1}{k_I}-C_{It}\right)l_{It}
\end{aligned}$$

It is essentially a linear equation system and can be easily solved.

$$\begin{bmatrix}\frac{B_{1t}}{k_1}-C_{1t} & \frac{B_{1t}}{k_2} & \cdots & \frac{B_{1t}}{k_I}\\ \frac{B_{2t}}{k_1} & \frac{B_{2t}}{k_2}-C_{2t} & \cdots & \frac{B_{2t}}{k_I}\\ \vdots & \vdots & \vdots & \vdots\\ \frac{B_{It}}{k_1} & \frac{B_{It}}{k_2} & \cdots & \frac{B_{It}}{k_I}-C_{It}\end{bmatrix}\begin{bmatrix}l_{1t}\\ l_{2t}\\ \vdots\\ l_{It}\end{bmatrix}=\begin{bmatrix}A_{1t}\\ A_{2t}\\ \vdots\\ A_{It}\end{bmatrix}$$

The optimal bidding decisions at time t are given as follows:

$$\begin{bmatrix} l_{1t} \\ l_{2t} \\ \vdots \\ l_{It} \end{bmatrix} = \begin{bmatrix} \frac{B_{1t}}{k_1} - C_{1t} & \frac{B_{1t}}{k_2} & \cdots & \frac{B_{1t}}{k_I} \\ \frac{B_{2t}}{k_1} & \frac{B_{2t}}{k_2} - C_{2t} & \cdots & \frac{B_{2t}}{k_I} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{B_{It}}{k_1} & \frac{B_{It}}{k_2} & \cdots & \frac{B_{It}}{k_I} - C_{It} \end{bmatrix}^{-1} \begin{bmatrix} A_{1t} \\ A_{2t} \\ \vdots \\ A_{It} \end{bmatrix}$$

where: $k_i = \beta_i, \ \forall i = 1, 2, \ldots, I$

Important observations:

1. The period t optimal bidding is determined by the parameters at time t only, it does not explicitly depend on the previous or future state variables (including initial or final states). But it does depend on other rival GENCOs' decision/bidding. So the optimal bidding for all GENCOs will be solved simultaneously. One can solve the T linear equation systems respectively in order to obtain the NEs over time periods T. The reason on the decoupling is that the costate variables and state variables are decoupled. Furthermore, costate variables are constant! The functional form of salvage value is a key point.

2. The salvage value of fuels does affect GENCOs' bidding strategies. The impacts are represented by the parameter γ_i, which is included into A_{it}, B_{it}, and C_{it}.

3. If state variables have more constraints like non-negativity, a similar decoupling situation will be expected.

4. The model can be easily to extend to include Take-or-Pay fuel (fix final state) contracts for some GENCOs. The impact of Take-or-Pay fuel contract on GENCO's optimal bidding can be evaluated.

5. The decoupling with respect to time is essentially because GENCO i considers its positions in both fuel and electricity markets in the proposed model. If only a single

electricity market is considered, a normal *Riccati-like* coupling equation will be expected [74]. It is more realistic to assume GENCOs to play in multiple markets than just one electricity market.

6. Another type of decoupling can be easily observed in an electricity market. Usually, an electricity market includes three major components: day-ahead, hourly-ahead, and real-time. A physical load is served by the scheduling activities of the tree markets. However, the three markets are run separately.

7. Besides the decoupling between the costate and state variables, costate variables are constant in the proposed model. It can be explained that in a day-ahead market, GENCOs only make estimation for the forward fuel price once when submitting a set of bids for multiple periods. Therefore, the costate variables, which are the shadow prices of fuel at each time period, are the same with the estimated forward price. The rules of a day-ahead market will result in a constant costate variable. However, the result does not mean that a multiple-period model is useless compared to a single-period. The estimated forward price of fuels does affect GENCOs' bidding strategies. The impacts are represented by the parameter, which is included into A_{it}, B_{it}, and C_{it}.

4.3 SFE with Transmission Congestion

In section 2.3, transmission network impacts are not incorporated in SFE bidding model. It is equivalent to say that all of the transmission lines have infinite large limits. In this case, prices are uniform throughout the entire network. However, physically some of the transmission lines have finite (or relative low) limits. If power flow hits the limit at one or more such transmission lines, prices will change at different locations that reflects the varying cost of transmitting one MW electricity to different locations. The difficulty incorporating transmission constraints is that the possible number of transmission congestions normally is huge given a large power system with many transmission lines. The situation of transmission congestions will vary discontinuously when GENCOs change their bidding decisions. Gedra proposed a general way to calculate DC optimal power flow sensitivity [68]. Lin Xu etc. [69] and Ross etc. [45] consider a SFE with transmission constrained based on DC OPF sensitivity. However, only a $k_{it}-$ parameterization model is studied. The information on transmission congestion is regarded exogenous. In other words, it is assumed that whether or not a line is going to be binding before GENCOs bidding into the market. This section will give a general derivation on SFE with transmission congestions as endogenous variables based on DC OPF sensitivity. $l_{it}-$ parameterization model will be used. However, usually multiple SFEs (pure and mixed) will exist, which is consistent to Ross's claim in [45]. In order to highlight transmission congestion effects, a single period model is considered only.

Notations:

Assumption: at most one GENCO and one load at one bus. If there are multiple generators or loads at one bus, an artificial generator or load could be defined.

n: index of buses; $n = 1, 2, \ldots, N$; N: the number of buses;

i: index of GENCOs; $i = 1, 2, \ldots, I$, I: the number of GENCOs;

j: index of demands; $j = 1, 2, \ldots, J$; J: the number of loads;

m: index of transmission lines; $m = 1, 2, \ldots, M$; M: the number of transmission lines;

r: index of congestions; $r = 1, 2, \ldots, R$; R: the number of congestions;

P_{it}: price at the bus corresponding to GENCO i;

P_{jt}: price at the bus corresponding to Load j;

Q_{jt}: demand of Load j at t;

q_{it}:: generation of GENCO i at t;

l_{it}: GENCO i's decision variable (the intercept of supply function, *i.e.* l_{it} – parameterization) at t;

A: node-arc incidence matrix, M by N;

B: B matrix in DC power flow, N by N;

D: susceptance matrix, M by M;

$\mathbf{q}_t$: a vector of generation q_{it} at t, I by 1;

$\mathbf{Q}_t$: a vector of demands Q_{jt} at t, J by 1;

$\boldsymbol{\theta}$: a vector of node phase angles, N by 1;

$\mathbf{p_b}$: a vector of branch power flows, M by 1;

$\boldsymbol{\lambda}$: a vector of dual variables corresponding to node power balance equations, N by 1 ;

$\boldsymbol{\mu}$: a vector of dual variables corresponding to branch flow equations, M by 1;

$\boldsymbol{\gamma}$: a vector of dual variables corresponding to transmission congestion lines, R by 1;

$\mathbf{p}$: a vector of node power injections, N by 1;

$\mathbf{p_{bmax}}$: a vector of branch power flow limits, M by 1;

GENCO i's supply function is assumed to be a linear increasing function and cost function is the integration of supply function:

$$P_{it} = l_{it} + k_i q_{it} \quad \left(i = 1,\ 2,\ \dots,\ I,\ \ k_i > 0 \right)$$
$$Cost_{it} = \int P_{it} dq_{it} = \frac{1}{2}\left(l_{it} + l_{it} + k_i q_{it} \right) q_{it} = l_{it} q_{it} + \frac{1}{2} k_i q_{it}^2 \ ;$$

The shaded area under a supply function in Fig. 23 is GENCO i's cost;

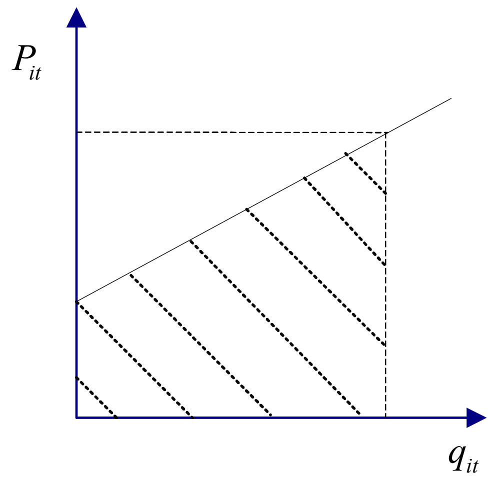

Figure 23: GENCO i's Supply Function and Cost

Load j's demand function is assumed to be a linear decreasing function and utility function is the integration of demand function:

$$P_{jt} = h_{jt} - g_j Q_{jt} \quad \left(j = 1,\ 2,\ \dots,\ J,\ \ g_j > 0 \right)$$
$$Utility_{jt} = \int P_{jt} dQ_{jt} = \frac{1}{2}\left(h_{jt} + h_{jt} - g_j Q_{jt} \right) Q_{jt} = h_{jt} Q_{jt} - \frac{1}{2} g_j Q_{jt}^2 \ ;$$

The shaded area under a demand function in Fig. 24 is Load j's utility. In a deregulated electricity market, LSEs are usually assumed to purchase energy on behalf of consumers. If retail market is perfectly competitive, i.e. consumers are able to choose LSEs freely, the demand of LSEs will reflect the true demand of consumers.

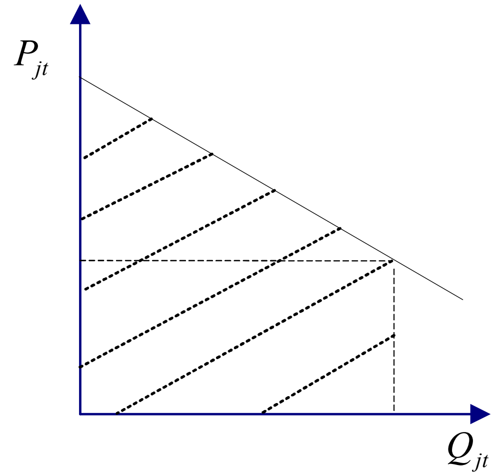

Figure 24: Load *j*'s Demand Function and Utility

Therefore, the total costs of *I* GENCOs and total utilities of *J* Loads are given below:

Total costs:

$$\sum_{i=1}^{I} Cost_{it} = \sum_{i=1}^{I} l_{it} q_{it} + \frac{1}{2}\sum_{i=1}^{I} k_i q_{it}^2 = [l_{1t} \quad l_{2t} \quad \cdots \quad l_{It}] \begin{bmatrix} q_{1t} \\ q_{2t} \\ \vdots \\ q_{It} \end{bmatrix} + \frac{1}{2}[q_{1t} \quad q_{2t} \quad \cdots \quad q_{It}] \begin{bmatrix} k_1 & & 0 \\ & \ddots & \\ 0 & & k_I \end{bmatrix} \begin{bmatrix} q_{1t} \\ q_{2t} \\ \vdots \\ q_{It} \end{bmatrix}$$

$$= \mathbf{l}_t^T \mathbf{q}_t + \frac{1}{2}\mathbf{q}_t^T \mathbf{k}_t \mathbf{q}_t$$

Total utilities

$$\sum_{j=1}^{J} Utility_{jt} = \sum_{j=1}^{J} h_j Q_{jt} - \frac{1}{2}\sum_{j=1}^{J} g_j Q_{jt}^2 = [h_{1t} \quad h_{2t} \quad \cdots \quad h_{Jt}] \begin{bmatrix} Q_{1t} \\ Q_{2t} \\ \vdots \\ Q_{Jt} \end{bmatrix} + \frac{1}{2}[Q_{1t} \quad Q_{2t} \quad \cdots \quad Q_{Jt}] \begin{bmatrix} g_1 & & 0 \\ & \ddots & \\ 0 & & g_J \end{bmatrix} \begin{bmatrix} Q_{1t} \\ Q_{2t} \\ \vdots \\ Q_{Jt} \end{bmatrix}$$

$$= \mathbf{h}_t^T \mathbf{Q}_t - \frac{1}{2}\mathbf{Q}_t^T \mathbf{g}_t \mathbf{Q}_t$$

Thus the second level problem (ISO maximizes social welfare) is defined as below:

$$\begin{aligned}
\max\quad & \mathbf{h}_t^T\mathbf{Q}_t - \frac{1}{2}\mathbf{Q}_t^T\mathbf{g}_t\mathbf{Q}_t - \mathbf{l}_t^T\mathbf{q}_t - \frac{1}{2}\mathbf{q}_t^T\mathbf{k}_t\mathbf{q}_t \\
s.t.\quad & \mathbf{p}=\mathbf{B\theta} \Rightarrow -\mathbf{p}+\mathbf{B\theta}=0 \\
& \mathbf{p_b} = (\mathbf{DA})\mathbf{\theta} \Rightarrow -\mathbf{p_b} + (\mathbf{DA})\mathbf{\theta} = 0 \\
& \mathbf{\text{-}p_{bmax}} \le \mathbf{p_b} \le \mathbf{p_{bmax}} \\
& q_{i\min} \le q_{it} \le q_{i\max} \\
& Q_{j\min} \le Q_{jt} \le Q_{j\max} \\
& \mathbf{p} = \mathbf{q}_t - \mathbf{Q}_t
\end{aligned}$$

In order to derive a compact model, it is necessary to define a Generator-Bus incidence matrix **E** and a Demand-Bus incidence matrix **F**, which relate buses to GENCOs or Loads:

Define a Generator-Bus incidence matrix **E** (*I* by *N*)

$$\mathbf{E} = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ I \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & N \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ & & & \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Define a Demand-Bus incidence matrix **F** (*J* by *N*)

$$\mathbf{F} = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ J \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & N \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ & & & \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Then the node net injection power flow vector can be expressed below:

$$\mathbf{p} = \mathbf{E}^T\mathbf{q}_t - \mathbf{F}^T\mathbf{Q}_t$$

The node power balance equation become:

$$-\mathbf{p}+\mathbf{B\theta}=0 \Rightarrow -\left(\mathbf{E}^T\mathbf{q}_t-\mathbf{F}^T\mathbf{Q}_t\right)+\mathbf{B\theta}=0$$

Define a matrix **S = DA**, the branch power flow equation become:

$$-\mathbf{p_b}+(\mathbf{DA})\mathbf{\theta}=0 \Rightarrow -\mathbf{p_b}+\mathbf{S\theta}=0$$

Furthermore, define a Congestion-Line incidence matrix **C** (*R* by *M*), which specifies the exact location where congestion occurs.

$$\mathbf{C}=\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ R \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & M \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & & & \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The Congestion-Line incidence matrix **C** will give us a lot of flexibility to deal with transmission congestions. When the congestion conditions change according to GENCOs' biddings, one just needs to simply modify the matrix **C** (add or delete or modify a specific row of matrix **C**). The method can handle all kinds of congestions scenarios.

The transmission congestion constraints are:

$$\mathbf{Cp_b}=\mathbf{Cp_{blimit}} \Rightarrow \mathbf{Cp_b}-\mathbf{Cp_{blimit}}=0 \quad \text{Where: } \mathbf{p_{blimit}}=\mathbf{p_{bmax}} \text{ or } -\mathbf{p_{bmax}}$$

Define a Lagrangian function as below:

$$\mathbf{L}=\mathbf{h}_t^T\mathbf{Q}_t-\frac{1}{2}\mathbf{Q}_t^T\mathbf{g}_t\mathbf{Q}_t-\mathbf{l}_t^T\mathbf{q}_t-\frac{1}{2}\mathbf{q}_t^T\mathbf{k}_t\mathbf{q}_t+\mathbf{\lambda}^T\left(\mathbf{B\theta}-\mathbf{E}^T\mathbf{q}_t+\mathbf{F}^T\mathbf{Q}_t\right)$$
$$+\mathbf{\mu}^T\left(\mathbf{S\theta}-\mathbf{p_b}\right)+\mathbf{\gamma}^T\left(\mathbf{Cp_b}-\mathbf{Cp_{blimit}}\right)$$

The first-order conditions are:

$$\frac{\partial \mathbf{L}}{\partial \mathbf{q}_t} = -\mathbf{l}_t^T - \mathbf{q}_t^T \mathbf{k}_t - \boldsymbol{\lambda}^T \mathbf{E}^T = 0 \Rightarrow \mathbf{l}_t + \mathbf{k}_t \mathbf{q}_t + \mathbf{E}\boldsymbol{\lambda} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \mathbf{Q}_t} = \mathbf{h}_t^T - \mathbf{Q}_t^T \mathbf{g}_t + \boldsymbol{\lambda}^T \mathbf{F}^T = 0 \Rightarrow \mathbf{h}_t - \mathbf{g}_t \mathbf{Q}_t + \mathbf{F}\boldsymbol{\lambda} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \boldsymbol{\theta}} = \boldsymbol{\lambda}^T \mathbf{B} + \boldsymbol{\mu}^T \mathbf{S} = 0 \Rightarrow \mathbf{B}^T \boldsymbol{\lambda} + \mathbf{S}^T \boldsymbol{\mu} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \mathbf{p_b}} = -\boldsymbol{\mu}^T + \boldsymbol{\gamma}^T \mathbf{C} = 0 \Rightarrow -\boldsymbol{\mu} + \mathbf{C}^T \boldsymbol{\gamma} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \boldsymbol{\lambda}} = \left(\mathbf{B}\boldsymbol{\theta} - \mathbf{E}^T \mathbf{q}_t + \mathbf{F}^T \mathbf{Q}_t\right)^T = 0 \Rightarrow \mathbf{B}\boldsymbol{\theta} - \mathbf{E}^T \mathbf{q}_t + \mathbf{F}^T \mathbf{Q}_t = 0$$

$$\frac{\partial \mathbf{L}}{\partial \boldsymbol{\mu}} = \left(\mathbf{S}\boldsymbol{\theta} - \mathbf{p_b}\right)^T = 0 \Rightarrow \mathbf{S}\boldsymbol{\theta} - \mathbf{p_b} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \boldsymbol{\gamma}} = \left(\mathbf{C}\mathbf{p_b} - \mathbf{C}\mathbf{p_{blimit}}\right)^T = 0 \Rightarrow \mathbf{C}\mathbf{p_b} - \mathbf{C}\mathbf{p_{blimit}} = 0$$

One can formulate the FOC as a linear equation system as below:

$$\begin{array}{c} \\ I \\ J \\ N \\ M \\ N \\ M \\ R \end{array}
\begin{array}{c} \begin{array}{ccccccc} I & J & N & M & N & M & R \end{array} \\
\begin{bmatrix} \mathbf{k}_t & 0 & 0 & 0 & \mathbf{E} & 0 & 0 \\ 0 & -\mathbf{g}_t & 0 & 0 & \mathbf{F} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{B}^T & \mathbf{S}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbf{I} & \mathbf{C}^T \\ -\mathbf{E}^T & \mathbf{F}^T & \mathbf{B} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{S} & -\mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C} & 0 & 0 & 0 \end{bmatrix} \end{array}
\begin{array}{c} \\ \begin{bmatrix} \mathbf{q}_t \\ \mathbf{Q}_t \\ \boldsymbol{\theta} \\ \mathbf{p_b} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \boldsymbol{\gamma} \end{bmatrix} \end{array}
\begin{array}{c} \\ = \end{array}
\begin{array}{c} \\ \begin{bmatrix} -\mathbf{l}_t \\ -\mathbf{h}_t \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{C}\mathbf{p_{blimit}} \end{bmatrix} \end{array} \tag{27}$$

Define

$$\mathbf{z} = \begin{bmatrix} \mathbf{q}_t \\ \mathbf{Q}_t \\ \boldsymbol{\theta} \\ \mathbf{p_b} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \boldsymbol{\gamma} \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} -\mathbf{l}_t \\ -\mathbf{h}_t \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{C}\mathbf{p_{blimit}} \end{bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} \mathbf{k}_t & 0 & 0 & 0 & \mathbf{E} & 0 & 0 \\ 0 & -\mathbf{g}_t & 0 & 0 & \mathbf{F} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{B}^T & \mathbf{S}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbf{I} & \mathbf{C}^T \\ -\mathbf{E}^T & \mathbf{F}^T & \mathbf{B} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{S} & -\mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C} & 0 & 0 & 0 \end{bmatrix}$$

We are only interested in the case that equation (27) has a unique solution. Therefore it is assumed that matrix $\mathbf{\Phi}$ is invertible in the rest of the study.

Define $\mathbf{W} = \mathbf{\Phi}^{-1}$, then

$$\mathbf{\Phi z} = \boldsymbol{\omega} \Rightarrow \mathbf{z}^* = \mathbf{\Phi}^{-1}\boldsymbol{\omega} = \mathbf{W}\boldsymbol{\omega} \tag{28}$$

Rewrite the matrix form of equation as below:

$$\mathbf{\Phi z} = \boldsymbol{\omega} \Rightarrow \begin{array}{c} \phi_{11}z_1 + \phi_{12}z_2 + \cdots + \phi_{1n}z_n = \omega_1 \\ \phi_{21}z_1 + \phi_{22}z_2 + \cdots + \phi_{2n}z_n = \omega_2 \\ \vdots \\ \phi_{v1}z_1 + \phi_{v2}z_2 + \cdots + \phi_{vn}z_n = \omega_v \\ \vdots \\ \phi_{V1}z_1 + \phi_{V2}z_2 + \cdots + \phi_{Vn}z_n = \omega_V \end{array}$$

One can calculate the sensitivity of the optimal solution with respect to a parameter ϕ_{vv}. Notice: ϕ_{vv} represents a slope parameter.

$$\frac{\partial(\mathbf{\Phi z})}{\partial \phi_{vv}} = 0 \Rightarrow \begin{array}{c} 0 + \phi_{11}\dfrac{\partial z_1}{\partial \phi_{vv}} + \phi_{12}\dfrac{\partial z_2}{\partial \phi_{vv}} + \cdots + \phi_{1n}\dfrac{\partial z_n}{\partial \phi_{vv}} = 0 \\ 0 + \phi_{21}\dfrac{\partial z_1}{\partial \phi_{vv}} + \phi_{22}\dfrac{\partial z_2}{\partial \phi_{vv}} + \cdots + \phi_{2n}\dfrac{\partial z_n}{\partial \phi_{vv}} = 0 \\ \vdots \\ z_v + \phi_{v1}\dfrac{\partial z_1}{\partial \phi_{vv}} + \phi_{v2}\dfrac{\partial z_2}{\partial \phi_{vv}} + \cdots + \phi_{vn}\dfrac{\partial z_n}{\partial \phi_{vv}} = 0 \\ \vdots \\ 0 + \phi_{V1}\dfrac{\partial z_1}{\partial \phi_{VV}} + \phi_{V2}\dfrac{\partial z_2}{\partial \phi_{VV}} + \cdots + \phi_{Vn}\dfrac{\partial z_n}{\partial \phi_{VV}} = 0 \end{array} \Rightarrow \mathbf{\Phi}\frac{\partial \mathbf{z}^*}{\partial \phi_{vv}} = -\begin{bmatrix} 0 \\ 0 \\ \vdots \\ z_v^* \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{\partial \mathbf{z}^*}{\partial \phi_{vv}} = -\mathbf{\Phi}^{-1}\mathbf{I}_v\mathbf{z}^* = -\mathbf{W}\mathbf{I}_v\mathbf{z}^*$$

Where: $\mathbf{I}_v$ is a matrix of the same dimension with $\mathbf{\Phi}$, only (v, v) element is 1, others are zeros.

One can calculate the sensitivity of the optimal solution with respect to a parameter ω_v. Notice: ω_v represents an intercept parameter.

$$\frac{\partial \mathbf{z}^*}{\partial \omega_v} = \frac{\partial(\mathbf{W}\boldsymbol{\omega})}{\partial \omega_v} = \mathbf{W}\frac{\partial \boldsymbol{\omega}}{\partial \omega_v} = \mathbf{W}\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \mathbf{W}\mathbf{1}_v$$

Where: $\mathbf{1}_v$ is a vector of the same number of row with $\boldsymbol{\Phi}$, only v element is 1, others are zeros.

Then one has $\frac{\partial q_{it}^*}{\partial k_i} = -W_{ii}q_{it}^*$, $\frac{\partial q_{it}^*}{\partial l_{it}} = -W_{ii}$, W_{jj} is the (j, j) element of $\mathbf{W}$. The result is given in [45] [69].

Then, let us consider the first level problem (GENCO maximizes its profit), the FOC is

$$\begin{aligned} &\frac{\partial \pi_{it}}{\partial k_i} = 0 \Rightarrow l_{it}\frac{\partial q_{it}^*}{\partial k_i} - \alpha_t\frac{\partial q_{it}^*}{\partial k_i} + q_{it}^{*2} + 2k_i q_{it}^*\frac{\partial q_{it}^*}{\partial k_i} - \beta_t q_{it}^*\frac{\partial q_{it}^*}{\partial k_i} = 0 \\ &\Rightarrow -l_{it}W_{ii}q_{it}^* + \alpha_t W_{ii}q_{it}^* + q_{it}^{*2} - 2k_i W_{ii}q_{it}^{*2} + \beta_t W_{ii}q_{it}^{*2} = 0 \\ &\Rightarrow \left(-l_{it}W_{ii} + \alpha_t W_{ii} + q_{it}^* - 2k_i W_{ii}q_{it}^* + \beta_t W_{ii}q_{it}^*\right)q_{it}^* = 0 \end{aligned} \qquad (29)$$

and

$$\begin{aligned} &\frac{\partial \pi_{it}}{\partial l_{it}} = 0 \Rightarrow l_{it}\frac{\partial q_{it}^*}{\partial l_{it}} - \alpha_t\frac{\partial q_{it}^*}{\partial l_{it}} + q_{it}^* + 2k_i q_{it}^*\frac{\partial q_{it}^*}{\partial l_{it}} - \beta_t q_{it}^*\frac{\partial q_{it}^*}{\partial l_{it}} = 0 \\ &\Rightarrow -l_{it}W_{ii} + \alpha_t W_{ii} + q_{it}^* - 2k_i W_{ii}q_{it}^* + \beta_t W_{ii}q_{it}^* = 0 \end{aligned} \qquad (30)$$

According to both of the two equations, it is easily to find that $\frac{\partial \pi_{it}}{\partial l_{it}} = 0 \Rightarrow \frac{\partial \pi_{it}}{\partial k_i} = 0$.

The fact is consistent with the observation in SFE without transmission constraints that one has to assign I unknowns first, then calculate the other I variables under equilibrium conditions because there are $2I$ unknowns $\left(l_{it}, k_i\right)$, but only I equations.

Finally, the optimal intercept parameters of supply function will be derived.

Revisit the equation (28)

$$\mathbf{z}^* = \mathbf{W}\boldsymbol{\omega}, \quad q_{it}^* = \left(\mathbf{z}^*\right)_i = \left(\mathbf{W}\boldsymbol{\omega}\right)_i = \left(\mathbf{W}\right)_i \boldsymbol{\omega}$$

One can partition $\mathbf{W}$ matrix into to 4 small block matrix and $\boldsymbol{\omega}$ vector into 2 vectors

$$\mathbf{W} = \begin{array}{c} \\ I \\ \\ \end{array}\!\!\begin{array}{c} \quad I \\ \begin{bmatrix} \mathbf{W}^{(1)} & \mathbf{W}^{(2)} \\ \mathbf{W}^{(3)} & \mathbf{W}^{(4)} \end{bmatrix} \end{array},$$

$$\boldsymbol{\omega} = \begin{array}{c} \\ I \\ \\ \\ \\ \\ \\ \\ \end{array}\!\!\begin{array}{c} 1 \\ \begin{bmatrix} -\mathbf{l}_t \\ -\mathbf{h}_t \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{CP}_{\mathbf{Blimit}} \end{bmatrix} \end{array} = \begin{bmatrix} \boldsymbol{\omega}^{(1)} \\ \boldsymbol{\omega}^{(2)} \end{bmatrix}, \quad \boldsymbol{\omega}^{(1)} = \left[-\mathbf{l}_t\right], \quad \boldsymbol{\omega}^{(2)} = \begin{bmatrix} -\mathbf{h}_t \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{CP}_{\mathbf{Blimit}} \end{bmatrix}$$

Then, for each equation i, one has:

$$q_{it}^* = \left(\mathbf{W}\right)_i \boldsymbol{\omega} = \left[\left(\mathbf{W}^{(1)}\right)_i \quad \left(\mathbf{W}^{(2)}\right)_i\right]\begin{bmatrix} \boldsymbol{\omega}^{(1)} \\ \boldsymbol{\omega}^{(2)} \end{bmatrix} = \left(\mathbf{W}^{(1)}\right)_i \boldsymbol{\omega}^{(1)} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)}$$

Substituting q_{it}^* into equation (30)

$$\frac{\partial \pi_{it}}{\partial l_{it}} = 0 \Rightarrow -l_{it}W_{ii} + \alpha_t W_{ii} + \left(1 - 2k_i W_{ii} + \beta_t W_{ii}\right) q_{it}^{*} = 0$$

$$\Rightarrow -l_{it}W_{ii} + \alpha_t W_{ii} + \left(1 - 2k_i W_{ii} + \beta_t W_{ii}\right)\left[\left(\mathbf{W}^{(1)}\right)_i \boldsymbol{\omega}^{(1)} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)}\right] = 0$$

$$\Rightarrow \frac{\alpha_t W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)} - \frac{W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} l_{it} + \left(\mathbf{W}^{(1)}\right)_i \boldsymbol{\omega}^{(1)} = 0$$

$$\Rightarrow \frac{\alpha_t W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)} - \frac{W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} l_{it} + \left(\mathbf{W}^{(1)}\right)_{i1}\left(-l_{1t}\right) + \left(\mathbf{W}^{(1)}\right)_{i2}\left(-l_{2t}\right)$$

$$+ \cdots + \left(\mathbf{W}^{(1)}\right)_{ii}\left(-l_{it}\right) + \cdots + \left(\mathbf{W}^{(1)}\right)_{II}\left(-l_{It}\right) = 0$$

$$\Rightarrow \frac{\alpha_t W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)} + W_{i1}\left(-l_{1t}\right) + W_{i2}\left(-l_{2t}\right)$$

$$+ \cdots + \left(W_{ii} + \frac{W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}}\right)\left(-l_{it}\right) + \cdots + W_{iI}\left(-l_{It}\right) = 0$$

$$\Rightarrow \begin{bmatrix} W_{i1} & W_{i2} & \cdots & W_{ii} + \frac{W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} & \cdots & W_{iI} \end{bmatrix} \begin{bmatrix} l_{1t} \\ l_{2t} \\ \vdots \\ l_{it} \\ \vdots \\ l_{It} \end{bmatrix} = \frac{\alpha_t W_{ii}}{1 - 2k_i W_{ii} + \beta_t W_{ii}} + \left(\mathbf{W}^{(2)}\right)_i \boldsymbol{\omega}^{(2)}$$

F

ollowing the same procedure for $\forall \quad i = 1, 2, \ldots, I$, one can get the linear equation system

$$\begin{bmatrix} W_{11} + \frac{W_{11}}{1-2k_1W_{11}+\beta_1W_{11}} & W_{12} & \cdots & W_{1i} & \cdots & W_{1I} \\ W_{21} & W_{22} + \frac{W_{22}}{1-2k_2W_{22}+\beta_2W_{22}} & \cdots & W_{2i} & \cdots & W_{2I} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ W_{i1} & W_{i2} & \cdots & W_{ii} + \frac{W_{ii}}{1-2k_iW_{ii}+\beta_iW_{ii}} & \cdots & W_{iI} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ W_{I1} & W_{I2} & \cdots & W_{Ii} & \cdots & W_{II} + \frac{W_{II}}{1-2k_IW_{II}+\beta_IW_{II}} \end{bmatrix} \begin{bmatrix} l_{1t} \\ l_{2t} \\ \vdots \\ l_{it} \\ \vdots \\ l_{It} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1 W_{11}}{1-2k_1W_{11}+\beta_1W_{11}} \\ \frac{\alpha_2 W_{22}}{1-2k_2W_{22}+\beta_2W_{22}} \\ \vdots \\ \frac{\alpha_i W_{ii}}{1-2k_iW_{ii}+\beta_iW_{ii}} \\ \vdots \\ \frac{\alpha_I W_{II}}{1-2k_IW_{II}+\beta_IW_{II}} \end{bmatrix} + \mathbf{W}^{(2)}\boldsymbol{\omega}^{(2)}$$

then

$$\begin{bmatrix} l_{1t} \\ l_{2t} \\ \vdots \\ l_{it} \\ \vdots \\ l_{It} \end{bmatrix} = \begin{bmatrix} W_{11} + \frac{W_{11}}{1-2k_1 W_{11} + \beta_1 W_{11}} & W_{12} & \cdots & W_{1i} & \cdots & W_{1I} \\ W_{21} & W_{22} + \frac{W_{22}}{1-2k_2 W_{22} + \beta_2 W_{22}} & \cdots & W_{2i} & \cdots & W_{2I} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ W_{i1} & W_{i2} & \cdots & W_{ii} + \frac{W_{ii}}{1-2k_i W_{ii} + \beta_i W_{ii}} & \cdots & W_{iI} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ W_{I1} & W_{I2} & \cdots & W_{Ii} & \cdots & W_{II} + \frac{W_{II}}{1-2k_I W_{II} + \beta_I W_{II}} \end{bmatrix}^{-1} \left(\begin{bmatrix} \frac{\alpha_1 W_{11}}{1-2k_1 W_{11} + \beta_1 W_{11}} \\ \frac{\alpha_2 W_{22}}{1-2k_2 W_{22} + \beta_2 W_{22}} \\ \vdots \\ \frac{\alpha_i W_{ii}}{1-2k_i W_{ii} + \beta_i W_{ii}} \\ \vdots \\ \frac{\alpha_I W_{II}}{1-2k_I W_{II} + \beta_I W_{II}} \end{bmatrix} + \mathbf{W}^{(2)} \boldsymbol{\omega}^{(2)} \right)$$

where: $k_i = \beta_i, \ \forall i = 1, 2, \ldots, I$

By solving the equation, the optimal bidding parameter at equilibrium will be obtained.

Important observation:

1. Assuming that there are R out of total M transmission lines that have transmission limits. However, the other $M - R$ transmission lines are assumed to have (relative) infinite large transmission limits. The result tells us that the SFE without congestion may or may not exist.

2. The existence of SFE with R, R-1, R-2, …, 1 lines congestion and uncongestion can be tested by checking whether the power flow on specific lines exceeds corresponding limits after the second level optimization problem without transmission congestions being solved.

3. If more than two pure strategies SFEs exist, there may be mixed strategies SFEs exist, too.

4. In practice, system operators would like to avoid transmission congestions. If transmission congestions happen, the system becomes insecure. Some small disturbances may cause a large system outage. A direct method is to upgrade transmission lines if there

is no uncongestion equilibrium existing in the original system. The result tells us that the corresponding transmission limits need to be increased to a certain level so that an uncongestion equilibrium exist.

5. The interesting fact is that system operators always prefer uncongestion or less congestion SFEs (if exist), however, GENCOs may not agree with them.

6. In reality, the existence of R lines congestion is also affected by price cap and production limit constraints. However, this model does not incorporate these factors.

General Procedures:

1. Given an electric power system, one can find some critical transmission lines where congestions often happen according to historical records. Assuming that there are such R transmission lines out of total M. However, the other $M-R$ transmission lines are assumed to have (relative) infinite large transmission limits.

2. Set $r = 0$.

3. Calculate the SFE with $R-r$ lines congestions.

4. After getting the optimal bids l_{it}^{*}, plug back into the second level problem without transmission congestions, check whether branch flows on the r lines is over transmission limits. If yes, the SFE with $(R-r)$-line congestion exist; Otherwise, does not exist.

5. $r = r + 1$, if $r > R$ go to step 6; Otherwise go to step 3.

6. Output these existing pure SFEs.

7. If more than one pure SFEs existing, then there may be some mixed SFEs exist. Calculate these mixed SFEs.

4.4 A Two-bus Case with Transmission Congestion

In order to show the feasibility of the proposed method, a two-bus case is constructed. There is only one line, the transmission limit is $\overline{p}_{12}$ & $\overline{p}_{21}$.

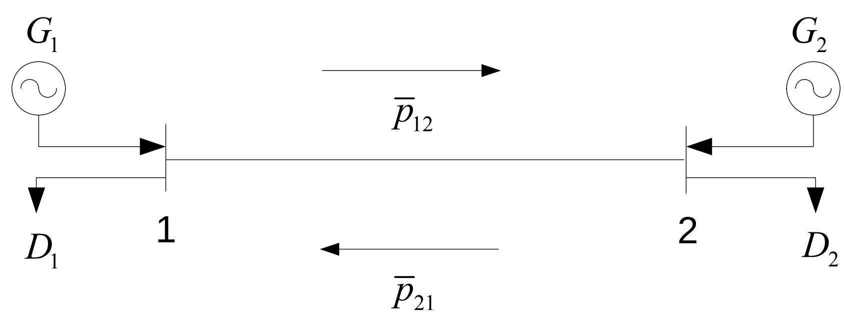

Figure 25: One Line Diagram of a Two-Bus System

GENCO i's supply function is assumed to be a linear increasing function and cost function is the integration of supply function:

$$P_{it} = l_{it} + k_i q_{it} \quad (i = 1,\ 2,\ \ k_i > 0)$$

$$Cost_{it} = \int P_{it} dq_{it} = \frac{1}{2}(l_{it} + l_{it} + k_i q_{it}) q_{it} = l_{it} q_{it} + \frac{1}{2} k_i q_{it}^2$$

Load j's demand function is assumed to be a linear decreasing function and utility function is the integration of demand function:

$$P_{jt} = h_{jt} - g_j Q_{jt} \quad (j = 1,\ 2,\ \ g_j > 0)$$

$$Utility_{jt} = \int P_{jt} dQ_{jt} = \frac{1}{2}(h_{jt} + h_{jt} - g_j Q_{jt}) Q_{jt} = h_{jt} Q_{jt} - \frac{1}{2} g_j Q_{jt}^2$$

Define a series of constants and matrix below:

$I = 2, J = 2, N = 2, M = 1, R = 1$

$$\mathbf{B}=\begin{bmatrix} y_{12} & -y_{12} \\ -y_{12} & y_{12} \end{bmatrix}, \quad \mathbf{D}=[y_{12}], \quad \mathbf{A}=[1 \quad -1], \quad \mathbf{S}=\mathbf{DA}=[y_{12} \quad -y_{12}],$$

$$\mathbf{E}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{F}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C}=[1]$$

The FOC is

$$\begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -g_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_{12} & -y_{12} & y_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_{12} & y_{12} & -y_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & y_{12} & -y_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -y_{12} & y_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{12} & -y_{12} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{1t} \\ q_{2t} \\ Q_{1t} \\ Q_{2t} \\ \theta_1 \\ \theta_2 \\ p_{12} \\ \lambda_1 \\ \lambda_2 \\ \mu \\ \gamma \end{bmatrix} = \begin{bmatrix} -l_{1t} \\ -l_{2t} \\ -h_{1t} \\ -h_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{p}_{12} \end{bmatrix}$$

In DC power flow, one has to arbitrarily choose a “reference bus” which has an angle value of 0; otherwise, the power flow solution is undetermined.

If bus 2 is assigned to be a reference bus, *i.e.* θ_2=0, one has to delete the variable θ_2 and the corresponding equation (the 6th column and row).

Therefore

$$\begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -g_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_{12} & -y_{12} & y_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_{12} & y_{12} & -y_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & y_{12} & -y_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -y_{12} & y_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{12} & -y_{12} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{1t} \\ q_{2t} \\ Q_{1t} \\ Q_{2t} \\ \theta_1 \\ \theta_2 \\ p_{12} \\ \lambda_1 \\ \lambda_2 \\ \mu \\ \gamma \end{bmatrix} = \begin{bmatrix} -l_{1t} \\ -l_{2t} \\ -h_{1t} \\ -h_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{p}_{12} \end{bmatrix}$$

Invert matrix $\mathbf{\Phi}$, one gets matrix $\mathbf{W}$. Partition matrix $\mathbf{W}$ into $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$.

Define some matrix as below:

$$\mathbf{W}^{(1)} = \begin{bmatrix} \frac{1}{g_1+k_1} & 0 \\ 0 & \frac{1}{g_2+k_2} \end{bmatrix},$$

$$\mathbf{W}^{(2)} = \begin{bmatrix} -\frac{1}{g_1+k_1} & 0 & 0 & 0 & -\frac{g_1}{g_1+k_1} & 0 & \frac{g_1}{g_1+k_1} & \frac{g_1}{g_1+k_1} \\ 0 & -\frac{1}{g_2+k_2} & 0 & 0 & 0 & -\frac{g_2}{g_2+k_2} & -\frac{g_2}{g_2+k_2} & -\frac{g_2}{g_2+k_2} \end{bmatrix}$$

$$\mathbf{\omega}^{(1)} = \begin{bmatrix} -l_{1t} \\ -l_{2t} \end{bmatrix}, \quad \mathbf{\omega}^{(2)} = \begin{bmatrix} -h_{1t} \\ -h_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{p}_{12} \end{bmatrix}$$

Then one has

$$\left[\mathbf{W}^{(1)}+\begin{bmatrix}\frac{W_{11}}{1-2k_1W_{11}+\beta_1W_{11}} & 0\\ 0 & \frac{W_{22}}{1-2k_2W_{22}+\beta_2W_{22}}\end{bmatrix}\right]\begin{bmatrix}l_{1t}^*\\ l_{2t}^*\end{bmatrix}=\begin{bmatrix}\frac{\alpha_1W_{11}}{1-2k_1W_{11}+\beta_1W_{11}}\\ \frac{\alpha_2W_{22}}{1-2k_2W_{22}+\beta_2W_{22}}\end{bmatrix}+\mathbf{W}^{(2)}\boldsymbol{\omega}^{(2)}$$

After solving the linear equation system, one gets the optimal bidding strategy:

$$l_{1t}^*=\frac{\left(-g_1k_1+g_1^2+\beta_1g_1\right)\overline{p}_{12}}{2g_1+\beta_1}+\frac{g_1h_1+\alpha_1k_1-h_1k_1+\alpha_1g_1+\beta_1h_1}{2g_1+\beta_1}$$
$$l_{2t}^*=\frac{\left(g_2k_2-g_2^2-\beta_2g_2\right)\overline{p}_{12}}{2g_2+\beta_2}+\frac{g_2h_2+\alpha_2k_2-h_2k_2+\alpha_2g_2+\beta_2h_2}{2g_2+\beta_2}$$

One can check if the congested equilibrium exists by plugging the optimal bids into the second level problem without congestion.

Specially, substituting $k_i=\beta_t$

$$l_{1t}^*=\frac{g_1^2\overline{p}_{12}}{2g_1+\beta_1}+\frac{g_1h_1+\alpha_1\beta_1+\alpha_1g_1}{2g_1+\beta_1}$$
$$l_{2t}^*=\frac{-g_2^2\overline{p}_{12}}{2g_2+\beta_2}+\frac{g_2h_2+\alpha_2\beta_2+\alpha_2g_2}{2g_2+\beta_2}$$

In order to confirm the result, an alternative derivation is shown below:

The first level problem is that ISO is to maximize the social welfare, *i.e.* maximize total utility – total cost

$$\max\ h_{1t}Q_{1t}-\frac{1}{2}g_1Q_{1t}^2+h_{2t}Q_{2t}-\frac{1}{2}g_2Q_{2t}^2-\left(l_{1t}q_{1t}+\frac{1}{2}k_1q_{1t}^2+l_{2t}q_{2t}+\frac{1}{2}k_2q_{2t}^2\right)$$

$$s.t.\ (\theta_1-\theta_2)y_{12}=q_{1t}-Q_{1t} \qquad (31)$$
$$(\theta_2-\theta_1)y_{12}=q_{2t}-Q_{2t} \qquad (32)$$
$$(\theta_1-\theta_2)y_{12}\le\overline{p}_{12} \qquad (33)$$
$$(\theta_2-\theta_1)y_{12}\le\overline{p}_{21} \qquad (34)$$

According to (31) & (32)

$$q_{1t}-Q_{1t}+q_{2t}-Q_{2t}=0$$

According to (31) & (33)

$$q_{1t}-Q_{1t}-\overline{p}_{12}+\frac{1}{2}\sigma^2=0$$

According to (32) & (34)

$$q_{2t}-Q_{2t}-\overline{p}_{21}+\frac{1}{2}\xi^2=0$$

Define the Lagrange function as below

$$L=h_{1t}Q_{1t}-\frac{1}{2}g_1Q_{1t}^2+h_{2t}Q_{2t}-\frac{1}{2}g_2Q_{2t}^2-l_{1t}q_{1t}-\frac{1}{2}k_1q_{1t}^2-l_{2t}q_{2t}-\frac{1}{2}k_2q_{2t}^2$$
$$+\lambda\left(q_{1t}-Q_{1t}+q_{2t}-Q_{2t}\right)+\mu\left(q_{1t}-Q_{1t}-\overline{p}_{12}+\frac{1}{2}\sigma^2\right)+\gamma\left(q_{2t}-Q_{2t}-\overline{p}_{21}+\frac{1}{2}\xi^2\right)$$

F.O.C

$$\frac{\partial L}{\partial q_{1t}}=-l_{1t}-k_1q_{1t}+\lambda+\mu=0,\quad \frac{\partial L}{\partial q_{2t}}=-l_{2t}-k_2q_{2t}+\lambda+\gamma=0$$
$$\frac{\partial L}{\partial Q_{1t}}=h_{1t}-g_1Q_{1t}-\lambda-\mu=0,\quad \frac{\partial L}{\partial Q_{2t}}=h_{2t}-g_2Q_{2t}-\lambda-\gamma=0$$
$$\frac{\partial L}{\partial \lambda}=q_{1t}-Q_{1t}+q_{2t}-Q_{2t}=0,\quad \frac{\partial L}{\partial \mu}=q_{1t}-Q_{1t}-\overline{p}_{12}+\frac{1}{2}\sigma^2=0$$
$$\frac{\partial L}{\partial \gamma}=q_{2t}-Q_{2t}-\overline{p}_{21}+\frac{1}{2}\xi^2=0,\quad \frac{\partial L}{\partial \sigma}=\mu\sigma=0,\quad \frac{\partial L}{\partial \xi}=\gamma\xi=0$$

Case 1: SFE with 1-line congestion

(a) $\overline{p}_{12}$ Congestion ($\sigma=0\ \&\ \xi\neq 0,\gamma=0$)

$$q_{1t}-Q_{1t}=\overline{p}_{12}$$
$$h_{2t}-g_2Q_{2t}=\lambda=l_{2t}+k_2q_{2t}$$
$$h_{1t}-g_1Q_{1t}=\lambda+\mu=l_{1t}+k_1q_{1t}$$
$$q_{1t}-Q_{1t}+q_{2t}-Q_{2t}=0$$

$$q_{1t}=\frac{h_{1t}-l_{1t}+g_1\overline{p}_{12}}{g_1+k_1},Q_{1t}=\frac{h_{1t}-l_{1t}-k_1\overline{p}_{12}}{g_1+k_1},$$
$$q_{2t}=\frac{h_{2t}-l_{2t}-g_2\overline{p}_{12}}{g_2+k_2},Q_{2t}=\frac{h_{2t}-l_{2t}+k_2\overline{p}_{12}}{g_2+k_2},$$
$$\lambda=\frac{\frac{l_{2t}}{k_2}+\frac{h_{2t}}{g_2}-\overline{p}_{12}}{\frac{1}{k_2}+\frac{1}{g_2}}=P_{2t},\lambda+\mu=\frac{\frac{l_{1t}}{k_1}+\frac{h_{1t}}{g_1}+\overline{p}_{12}}{\frac{1}{k_1}+\frac{1}{g_1}}=P_{1t}$$

Congestion equilibrium

For GENCO i:

$$\max \quad P_{it}q_{it}-Cost_{it}(q_{it})$$

It is a concave function w.r.t. l_{it}

$$\pi_{1t}=\frac{\frac{l_{1t}}{k_1}+\frac{h_{1t}}{g_1}+\overline{p}_{12}}{\frac{1}{k_1}+\frac{1}{g_1}}\left(\frac{h_{1t}-l_{1t}+g_1\overline{p}_{12}}{g_1+k_1}\right)-\alpha_1\left(\frac{h_{1t}-l_{1t}+g_1\overline{p}_{12}}{g_1+k_1}\right)-\frac{1}{2}\beta_1\left(\frac{h_{1t}-l_{1t}+g_1\overline{p}_{12}}{g_1+k_1}\right)^2$$

$$\pi_{2t}=\frac{\frac{l_{2t}}{k_2}+\frac{h_{2t}}{g_2}-\overline{p}_{12}}{\frac{1}{k_2}+\frac{1}{g_2}}\left(\frac{h_{2t}-l_{2t}-g_2\overline{p}_{12}}{g_2+k_2}\right)-\alpha_2\left(\frac{h_{2t}-l_{2t}-g_2\overline{p}_{12}}{g_2+k_2}\right)-\frac{1}{2}\beta_2\left(\frac{h_{2t}-l_{2t}-g_2\overline{p}_{12}}{g_2+k_2}\right)^2$$

$$\frac{\partial^2\pi_{1t}}{\partial l_{1t}^2}=-\frac{2g_1+\beta_1}{(g_1+k_1)^2}<0$$
$$\frac{\partial^2\pi_{2t}}{\partial l_{2t}^2}=-\frac{2g_2+\beta_2}{(g_2+k_2)^2}<0$$

$$l_{1t}^*=\frac{\left(-g_1k_1+g_1^2+\beta_1g_1\right)\overline{p}_{12}}{2g_1+\beta_1}+\frac{g_1h_1+\alpha_1k_1-h_1k_1+\alpha_1g_1+\beta_1h_1}{2g_1+\beta_1}$$
$$l_{2t}^*=\frac{\left(g_2k_2-g_2^2-\beta_2g_2\right)\overline{p}_{12}}{2g_2+\beta_2}+\frac{g_2h_2+\alpha_2k_2-h_2k_2+\alpha_2g_2+\beta_2h_2}{2g_2+\beta_2}$$

The result is the same with the previous one!

The second order condition requires that $\mu \le 0$, therefore

$$\mu = P_{1t} - P_{2t} = \frac{\frac{l_{1t}}{k_1} + \frac{h_{1t}}{g_1} + \overline{p}_{12}}{\frac{1}{k_1} + \frac{1}{g_1}} - \frac{\frac{l_{2t}}{k_2} + \frac{h_{2t}}{g_2} - \overline{p}_{12}}{\frac{1}{k_2} + \frac{1}{g_2}} \le 0$$

$$l_{1t} \le \frac{\left(\frac{1}{k_1} + \frac{1}{g_1}\right)\frac{1}{k_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} l_{2t} + \frac{\frac{1}{k_1}\frac{h_{2t}}{g_2} - \frac{1}{k_2}\frac{h_{1t}}{g_1} + \frac{h_{2t} - h_{1t}}{g_1 g_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} - \frac{\overline{p}_{12}\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}\right)}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}}$$

One can check if the optimal bids satisfy the condition.

The profits of two GENCOS are:

$$\pi_{1t}^* = \frac{0.5\left(h_1 + g_1\overline{p}_{12} - \alpha_1\right)^2}{2g_1 + \beta_1}$$

$$\pi_{2t}^* = \frac{0.5\left(h_2 - g_2\overline{p}_{12} - \alpha_2\right)^2}{2g_2 + \beta_2}$$

Which is not dependent on k_1&k_2

(b) $\overline{p}_{21}$ Congestion ($\sigma \ne 0, \mu = 0 \,\&\, \xi = 0$)

$$q_{2t} - Q_{2t} = \overline{p}_{21}$$

$$h_{1t} - g_1 Q_{1t} = \lambda = l_{1t} + k_1 q_{1t}$$

$$h_{2t} - g_2 Q_{2t} = \lambda + \gamma = l_{2t} + k_2 q_{2t}$$

$$q_{1t} - Q_{1t} + q_{2t} - Q_{2t} = 0$$

$$q_{1t}=\frac{h_{1t}-l_{1t}-g_1\overline{p}_{21}}{g_1+k_1},Q_{1t}=\frac{h_{1t}-l_{1t}+k_1\overline{p}_{21}}{g_1+k_1},$$
$$q_{2t}=\frac{h_{2t}-l_{2t}+g_2\overline{p}_{21}}{g_2+k_2},Q_{2t}=\frac{h_{2t}-l_{2t}-k_2\overline{p}_{21}}{g_2+k_2},$$
$$\lambda=\frac{\frac{l_{1t}}{k_1}+\frac{h_{1t}}{g_1}-\overline{p}_{21}}{\frac{1}{k_1}+\frac{1}{g_1}}=P_{1t},\lambda+\gamma=\frac{\frac{l_{2t}}{k_2}+\frac{h_{2t}}{g_2}+\overline{p}_{21}}{\frac{1}{k_2}+\frac{1}{g_2}}=P_{2t}$$

Congestion equilibrium

For GENCO i:

$$\max \quad P_{it}q_{it}-Cost_{it}(q_{it})$$

It is a concave function w.r.t. l_{it}

$$\pi_{1t}=\frac{\frac{l_{1t}}{k_1}+\frac{h_{1t}}{g_1}-\overline{p}_{21}}{\frac{1}{k_1}+\frac{1}{g_1}}\left(\frac{h_{1t}-l_{1t}-g_1\overline{p}_{21}}{g_1+k_1}\right)-\alpha_1\left(\frac{h_{1t}-l_{1t}-g_1\overline{p}_{21}}{g_1+k_1}\right)-\frac{1}{2}\beta_1\left(\frac{h_{1t}-l_{1t}-g_1\overline{p}_{21}}{g_1+k_1}\right)^2$$

$$\pi_{2t}=\frac{\frac{l_{2t}}{k_2}+\frac{h_{2t}}{g_2}+\overline{p}_{21}}{\frac{1}{k_2}+\frac{1}{g_2}}\left(\frac{h_{2t}-l_{2t}+g_2\overline{p}_{21}}{g_2+k_2}\right)-\alpha_2\left(\frac{h_{2t}-l_{2t}+g_2\overline{p}_{21}}{g_2+k_2}\right)-\frac{1}{2}\beta_2\left(\frac{h_{2t}-l_{2t}+g_2\overline{p}_{21}}{g_2+k_2}\right)^2$$

$$\frac{\partial^2\pi_{1t}}{\partial l_{1t}^2}=-\frac{2g_1+\beta_1}{(g_1+k_1)^2}<0$$
$$\frac{\partial^2\pi_{2t}}{\partial l_{2t}^2}=-\frac{2g_2+\beta_2}{(g_2+k_2)^2}<0$$

$$l_{1t}^*=\frac{\left(g_1k_1-g_1^2-\beta_1g_1\right)\overline{p}_{21}}{2g_1+\beta_1}+\frac{g_1h_1+\alpha_1k_1-h_1k_1+\alpha_1g_1+\beta_1h_1}{2g_1+\beta_1}$$
$$l_{2t}^*=\frac{\left(-g_2k_2+g_2^2+\beta_2g_2\right)\overline{p}_{21}}{2g_2+\beta_2}+\frac{g_2h_2+\alpha_2k_2-h_2k_2+\alpha_2g_2+\beta_2h_2}{2g_2+\beta_2}$$

The second order condition requires that $\gamma \le 0$, therefore

$$\gamma = P_{2t} - P_{1t} = \frac{\frac{l_{2t}}{k_2} + \frac{h_{2t}}{g_2} + \overline{p}_{21}}{\frac{1}{k_2} + \frac{1}{g_2}} - \frac{\frac{l_{1t}}{k_1} + \frac{h_{1t}}{g_1} - \overline{p}_{21}}{\frac{1}{k_1} + \frac{1}{g_1}} \le 0$$

$$l_{1t} \ge \frac{\left(\frac{1}{k_1} + \frac{1}{g_1}\right)\frac{1}{k_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} l_{2t} + \frac{\frac{1}{k_1}\frac{h_{2t}}{g_2} - \frac{1}{k_2}\frac{h_{1t}}{g_1} + \frac{h_{2t} - h_{1t}}{g_1 g_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} + \frac{\overline{p}_{21}\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}\right)}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}}$$

One can check if the optimal bids satisfy the condition.

The profits of two GENCOS are:

$$\pi_{1t}^* = \frac{0.5\left(h_1 - g_1\overline{p}_{21} - \alpha_1\right)^2}{2g_1 + \beta_1}$$

$$\pi_{2t}^* = \frac{0.5\left(h_2 + g_2\overline{p}_{21} - \alpha_2\right)^2}{2g_2 + \beta_2}$$

Which is not dependent on k_1&k_2

Case 2: SFE with uncongestion ($\mu = 0$ & $\gamma = 0$)

$$h_{1t} - g_1 Q_{1t} = \lambda \to P_{1t} \qquad h_{2t} - g_2 Q_{2t} = \lambda \to P_{2t}$$

$$l_{1t} + k_1 q_{1t} = \lambda \to P_{1t} \qquad l_{2t} + k_2 q_{2t} = \lambda \to P_{2t}$$

$$\therefore P_{1t} = P_{2t}$$

$$Q_{1t} = \frac{h_{1t} - \lambda}{g_1} \qquad q_{1t} = \frac{\lambda - l_{1t}}{k_1}$$

$$Q_{2t} = \frac{h_{2t} - \lambda}{g_2} \qquad q_{2t} = \frac{\lambda - l_{2t}}{k_2}$$

$$\frac{\lambda - l_{1t}}{k_1} + \frac{\lambda - l_{2t}}{k_2} - \frac{h_{1t} - \lambda}{g_1} - \frac{h_{2t} - \lambda}{g_2} = 0$$

$$\lambda = \frac{\frac{l_{1t}}{k_1} + \frac{l_{2t}}{k_2} + \frac{h_{1t}}{g_1} + \frac{h_{2t}}{g_2}}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}} = P_{1t} = P_{2t}$$

No congestion requires that

1. $q_{1t} - Q_{1t} < \overline{p}_{12}$

$$q_{1t} - Q_{1t} < \overline{p}_{12}$$
$$\frac{\lambda}{k_1} - \frac{l_{1t}}{k_1} - \frac{h_{1t}}{g_1} + \frac{\lambda}{g_1} < \overline{p}_{12}$$
$$\left(\frac{1}{k_1} + \frac{1}{g_1}\right)\frac{\frac{l_{1t}}{k_1} + \frac{l_{2t}}{k_2} + \frac{h_{1t}}{g_1} + \frac{h_{2t}}{g_2}}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}} - \frac{l_{1t}}{k_1} - \frac{h_{1t}}{g_1} < \overline{p}_{12}$$
$$l_{1t} > \frac{\left(\frac{1}{k_1} + \frac{1}{g_1}\right)\frac{1}{k_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} l_{2t} + \frac{\frac{1}{k_1}\frac{h_{2t}}{g_2} - \frac{1}{k_2}\frac{h_{1t}}{g_1} + \frac{h_{2t} - h_{1t}}{g_1 g_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} - \frac{\overline{p}_{12}\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}\right)}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} \quad (35)$$

2. $q_{2t} - Q_{2t} < \overline{p}_{21}$

$$q_{2t} - Q_{2t} < \overline{p}_{21}$$
$$\frac{\lambda}{k_2} - \frac{l_{2t}}{k_2} - \frac{h_{2t}}{g_2} + \frac{\lambda}{g_2} < \overline{p}_{21}$$
$$\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{\frac{l_{1t}}{k_1} + \frac{l_{2t}}{k_2} + \frac{h_{1t}}{g_1} + \frac{h_{2t}}{g_2}}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}} - \frac{l_{2t}}{k_2} - \frac{h_{2t}}{g_2} < \overline{p}_{21}$$
$$l_{1t} < \frac{\left(\frac{1}{k_1} + \frac{1}{g_1}\right)\frac{1}{k_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} l_{2t} + \frac{\frac{1}{k_1}\frac{h_{2t}}{g_2} - \frac{1}{k_2}\frac{h_{1t}}{g_1} + \frac{h_{2t} - h_{1t}}{g_1 g_2}}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} + \frac{\overline{p}_{21}\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{g_1} + \frac{1}{g_2}\right)}{\left(\frac{1}{k_2} + \frac{1}{g_2}\right)\frac{1}{k_1}} \quad (36)$$

Equation (35) & (36) define a feasible region for a SFE with uncongestion to exist. Fig. 26 shows the feasible region as a shaded area.

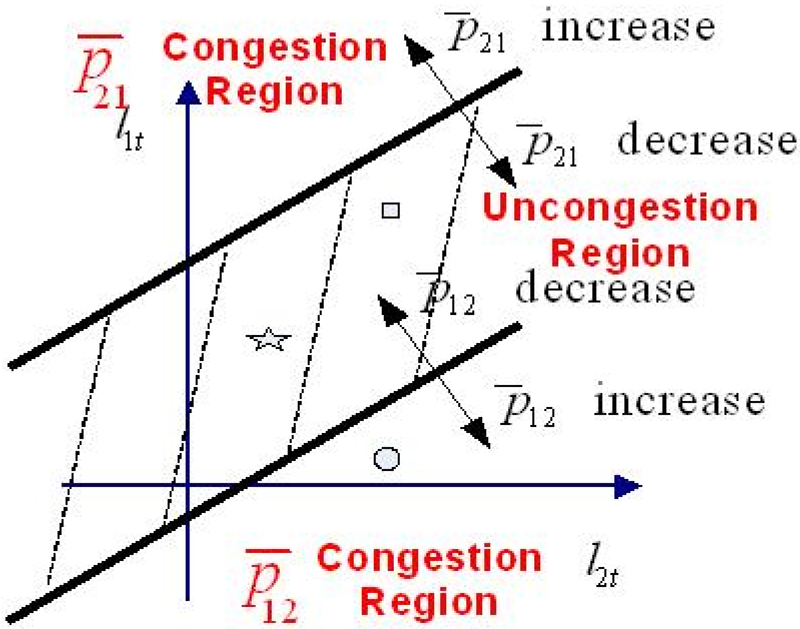

Figure 26: Feasible Region of Possible Equilibria

Where:

○ **unconstrained equilibrium**

☆ $\overline{p}_{12}$ **constrained equilibrium**

□ $\overline{p}_{21}$ **constrained equilibrium**

Uncongestion equilibrium

Assume $Cost_{it} = \alpha_i q_{it} + \frac{1}{2}\beta_i q_{it}^2$

For GENCO *i*:

max $P_{it} q_{it} - Cost_{it}(q_{it})$

It is a concave function w.r.t. l_{it}

$$\pi_{it} = \frac{\frac{l_{1t}}{k_1}+\frac{l_{2t}}{k_2}+\frac{h_{1t}}{g_1}+\frac{h_{2t}}{g_2}}{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{g_1}+\frac{1}{g_2}}\left(\frac{1}{k_i}\frac{\frac{l_{1t}}{k_1}+\frac{l_{2t}}{k_2}+\frac{h_{1t}}{g_1}+\frac{h_{2t}}{g_2}}{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{g_1}+\frac{1}{g_2}}-\frac{l_{it}}{k_i}\right)$$
$$-\alpha_i\left(\frac{1}{k_i}\frac{\frac{l_{1t}}{k_1}+\frac{l_{2t}}{k_2}+\frac{h_{1t}}{g_1}+\frac{h_{2t}}{g_2}}{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{g_1}+\frac{1}{g_2}}-\frac{l_{it}}{k_i}\right)-\frac{1}{2}\beta_i\left(\frac{1}{k_i}\frac{\frac{l_{1t}}{k_1}+\frac{l_{2t}}{k_2}+\frac{h_{1t}}{g_1}+\frac{h_{2t}}{g_2}}{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{g_1}+\frac{1}{g_2}}-\frac{l_{it}}{k_i}\right)^2$$

$$\frac{\partial^2\pi_{1t}}{\partial l_{1t}^2} = -\frac{(g_1g_2+k_2g_1+k_2g_2)(2k_2g_1g_2+\beta_1g_1g_2+\beta_1k_2g_1+\beta_1k_2g_2)}{(k_2g_1g_2+k_1g_1g_2+k_1k_2g_1+k_1k_2g_2)^2}<0$$
$$\frac{\partial^2\pi_{2t}}{\partial l_{2t}^2} = -\frac{(g_1g_2+k_1g_1+k_1g_2)(2k_1g_1g_2+\beta_2g_1g_2+\beta_2k_1g_1+\beta_2k_1g_2)}{(k_2g_1g_2+k_1g_1g_2+k_1k_2g_1+k_1k_2g_2)^2}<0$$

$$\left.\begin{aligned}\frac{\partial\pi_{1t}}{\partial l_{1t}}=0\\ \frac{\partial\pi_{2t}}{\partial l_{2t}}=0\end{aligned}\right\}\Rightarrow\begin{aligned}l_{1t}^*&=l_{1t}^*(h_{1t},h_{2t},k_1,k_2,g_1,g_2,\alpha_1,\alpha_2,\beta_1,\beta_2)\\ l_{2t}^*&=l_{2t}^*(h_{1t},h_{2t},k_1,k_2,g_1,g_2,\alpha_1,\alpha_2,\beta_1,\beta_2)\end{aligned}$$

The profits of two GENCOS are:

$$\pi_{1t}^* = \pi_{1t}^*(h_{1t},h_{2t},k_1,k_2,g_1,g_2,\alpha_1,\alpha_2,\beta_1,\beta_2)$$
$$\pi_{2t}^* = \pi_{2t}^*(h_{1t},h_{2t},k_1,k_2,g_1,g_2,\alpha_1,\alpha_2,\beta_1,\beta_2)$$

One can substitute $k_i=\beta_t$ into the equation above.

First, according to Fig. 26, it is observed that the uncongested equilibrium only exists inside the shaded area. If the shaded region is too small, *i.e.* the transmission limits are too low, the equilibrium may not exist. Therefore, the model is very instructive for transmission planning by ISO and Transmission Owner because it can provide a quantitive measurement for transmission line upgrade. This model indicates that it is necessary to consider market participants' strategic behaviors as well as the traditional reliability and economic criteria during the process of merchant transmission planning.

Second, since multiple equilibria may exist (the congested equilibrium may have a larger possibility), the ISO or policy maker should design some economic mechanism (market rules) to induce the market equilibrium from congested to uncongested as long as it exists.

Furthermore, one can formulate a matrix game as shown in the Table 8 to find all of pure and mixed strategy SFEs for the two-GENCO example. All of the three pure SFEs are assumed to exist. If the three SFEs are Pareto-Non-comparable, one can find a mixed strategy based on the specific parameters.

Table 8: A Matrix Game for a 2-GENCO Case

GENCO1 / GENCO2	Congestion SFE1	Congestion SFE2	Uncongestion SFE
Congestion SFE1	$\pi_{1t}^* = \frac{0.5(h_1 + g_1\bar{p}_{12} - \alpha_1)^2}{2g_1 + \beta_1}$ $\pi_{2t}^* = \frac{0.5(h_2 - g_2\bar{p}_{12} - \alpha_2)^2}{2g_2 + \beta_2}$		
Congestion SFE2		$\pi_{1t}^* = \frac{0.5(h_1 - g_1\bar{p}_{21} - \alpha_1)^2}{2g_1 + \beta_1}$ $\pi_{2t}^* = \frac{0.5(h_2 + g_2\bar{p}_{21} - \alpha_2)^2}{2g_2 + \beta_2}$	
Uncongestion SFE			$\pi_{1t}^* = \pi_{1t}^*(h_{it}, k_i, g_i, \alpha_i, \beta_i)$ $\pi_{2t}^* = \pi_{2t}^*(h_{it}, k_i, g_i, \alpha_i, \beta_i)$

CHAPTER 5. LEARNING ALGORITHMS

5.1 Introduction

Two SFE models are derived in Chapter Four theoretically. The pros of these analytical models are the predictability because of well-defined mathematical foundation. However, these models have been simplified according to many artificial assumptions. Some of them may not be true in practice. For example: if an assumption of linear supply function is relaxed, *i.e.* allowing piecewise linear or more general functional forms, it is normally impossible to derive analytical solutions. On the contrary, Chapter Five is trying to model GENCOs as adaptive agents, who have learning ability and are able to adjust strategies based on past experiences. Some learning algorithms are introduced to derive GENCOs' bidding strategies numerically. In this stage, only problems with single period linear SFE without transmission congestion are discussed.

Some good candidates of learning algorithms are ALIFE techniques. They are also called stochastic optimization in the arena of optimization. In order to show the feasibility of these stochastic optimization techniques, an EDC problem with CC units is solved by GA, EP, PS, and MILP firstly. Furthermore, the trajectory of each artificial life technique is shown and compared generation-by-generation.

5.2 EDC by Artificial Life Techniques

The three stochastic optimization techniques are numerically tested on two EDC cases.

The test system in Case 1 consists of two CC units. Cost curves of the two units are represented by fourth-order polynomial functions given by equation (19) in section 3.3. The associated incremental costs are non-monotonically increasing. Stationary points of the objective function may be local minimum, local maximum, or saddle points.

In Case 2, a system with 12 thermal units and 2 CC units is tested at the demand from 1200 MW to 3000 MW. A comparison between MILP (II) and stochastic techniques is presented.

5.2.1 Case 1

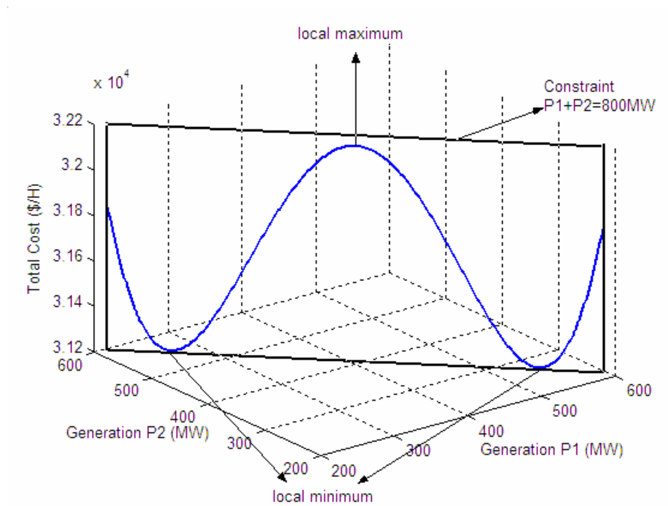

Figure 27: The Set of Intersection Points

Case 1 comprises two identical CC units at a demand of 800 MW. The problem formulation is given in (20)&(21). Fig. 27 shows the set of intersection points of the objective function surface and the constraint plane in a 3-D fashion. These intersection points form a smooth curve. It is clearly observed that there are two local minima and one symmetric local maximum on the curve.

In order to compare GA, EP, and PS on the same basis, the population size is set equal to 16, and the number of generations is set equal to 50. The optimal solution of each algorithm is summarized in Table 9:

Table 9: The Optimal Solution of Each Technique for Case 1

	Pop Size = 16		# of Gens = 50	
	CC unit1 (MW)	CC unit2 (MW)	Generation (MW)	Total Cost ($/H)
GA	560	240	800	31888
EP	528.75	271.25	800	31544
PS	510	290	800	31460
OPTIMAL	541	259	800	31221

Comparing the total costs in column 5, all of the three algorithms give good approximations of the optimal solution. The maximal relative error (GA) is only 2.14%. The errors are not totally from the algorithms since they partially depend on the accuracy of curve fitting (refer to (19)).

Fig. 28, 29, & 30 show the searching trajectories generation-by-generation of each technique. The velocity of PS contains more information than GA and EP. Specially, each individual particle has a memory of its own optimal value and the optimal value until the current generation. So all of the particles move in a fashion of the least randomness among the three algorithms.

EP makes use of a Gaussian random perturbation term to update each generation. EP does not apply local information; however, each generation does share a common knowledge implicitly. The mutation factor/standard deviation is proportional to the ratio between individual cost and the least cost up to now. The bigger the ratio, the higher the possibility of greater changing and vice versa.

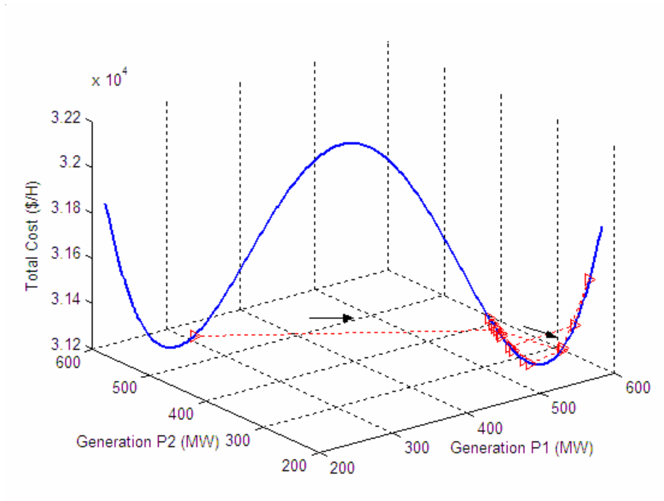

Figure 28: The Trajectory of GA

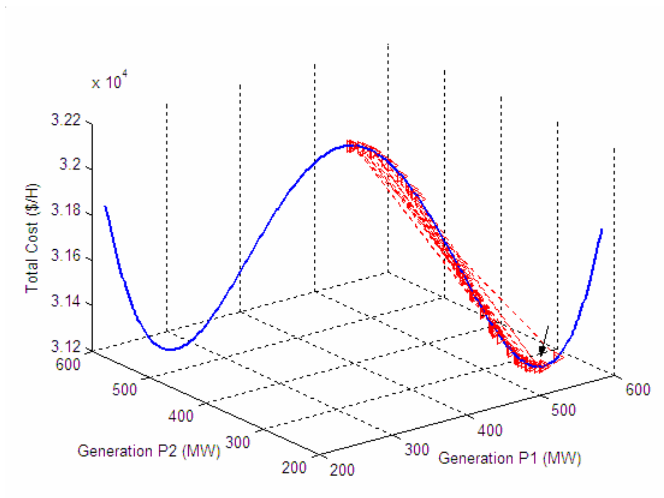

Figure 29: The Trajectory of EP

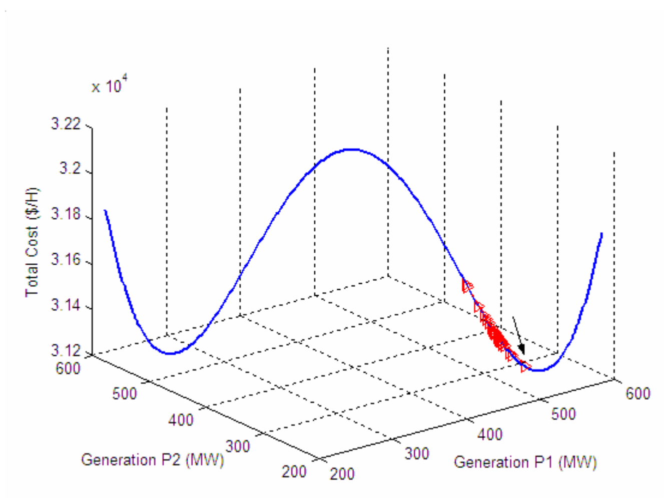

Figure 30: The Trajectory of PS

GA employs neither global nor local historical information. Crossover and mutation operators bring in a new sense of change in the searching direction through an essentially random way. Therefore, GA is able to search the biggest solution space of the three algorithms. [15]

5.2.2 Case 2

In this case, there are twelve thermal units and two identical CC units. The CC unit data are the same with Case 1. Assume both of CC units operate at state 4. Fig. 31 shows the system lambda curve of twelve thermal units. The curve is of course non-decreasing. There are two sudden jumps when demand is equal to 2272 and 2470. That is because some units operate at their maximal outputs and others operates at minimal, i.e. no unit is regulating. If lambda belongs to the ranges [10.02, 43.30] or [82.59, 87.30], the total generation does not vary with the changing of lambda.

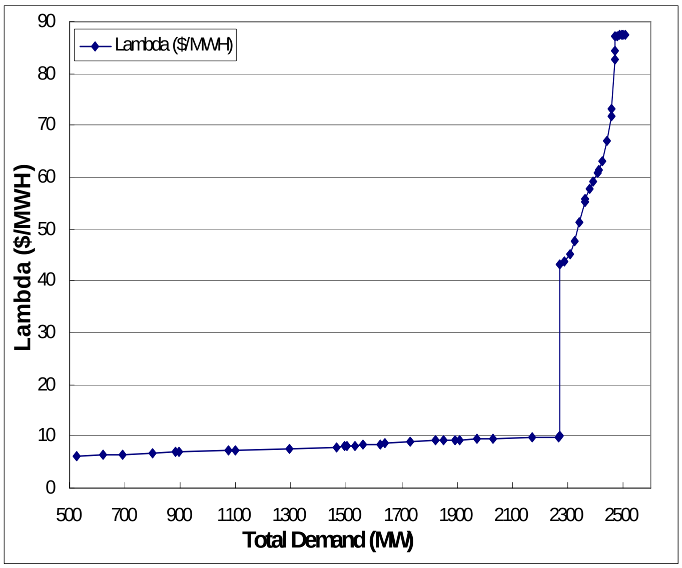

Figure 31: System Lambda Curve of Twelve Thermal Units

In order to do a comprehensive comparison, a MILP formulation (II) is implemented [16]. The MILP solutions are assumed as the true optimal solutions. Of course, the assumption may not be the truth for all cases. Then all of the four algorithms are run from demand is equal to 1200 to 3000 MW. The total cost for each demand and the relative error compared with MILP are listed in Table 10.

Table 10: The Comparison between MILP and GA/EP/PS for Case 2

Demand	GA		EP		PS		MILP
1200	30312	0.0030	30787	0.0187	30225	0.0001	30222
1300	30922	0.0004	31185	0.0089	30923	0.0005	30909
1400	31725	0.0035	32273	0.0208	31634	0.0006	31615
1500	32412	0.0026	32856	0.0163	32372	0.0014	32328
1600	33133	0.0020	33490	0.0128	33105	0.0011	33067
1700	33900	0.0026	35098	0.0380	33831	0.0005	33813
1800	34595	0.0003	35404	0.0237	34585	0.0000	34585
1900	35390	0.0005	36185	0.0229	35374	0.0000	35374
2000	36413	0.0055	37794	0.0436	36223	0.0002	36214
2100	37182	0.0023	38289	0.0322	37095	0.0000	37095
2200	38126	0.0038	39410	0.0376	37990	0.0002	37982
2300	39103	0.0052	38942	0.0011	38902	0.0001	38899
2400	40073	0.0056	40646	0.0200	39850	0.0000	39849
2500	40908	0.0023	41536	0.0177	40813	0.0000	40813
2600	41798	0.0003	42403	0.0147	41787	0.0000	41787
2700	43339	0.0005	44898	0.0365	43316	0.0000	43316
2800	46728	0.0285	47516	0.0459	45432	0.0000	45432
2900	50684	0.0559	48762	0.0159	48000	0.0000	48000
3000	55614	0.0794	51995	0.0091	51524	0.0000	51524
Avg Err	1.07%		2.3%		0.03%		

Fig. 32 shows a histogram of total costs of MILP and GA/EP/PS (population size = 128, generations = 100). It is observed that PS gives a very good approximation; the total cost is only 0.03% higher than optimal solution averagely. GA's performance is good except when demand is higher than 2800. EP produces relative lager errors in the middle of the range of demands.

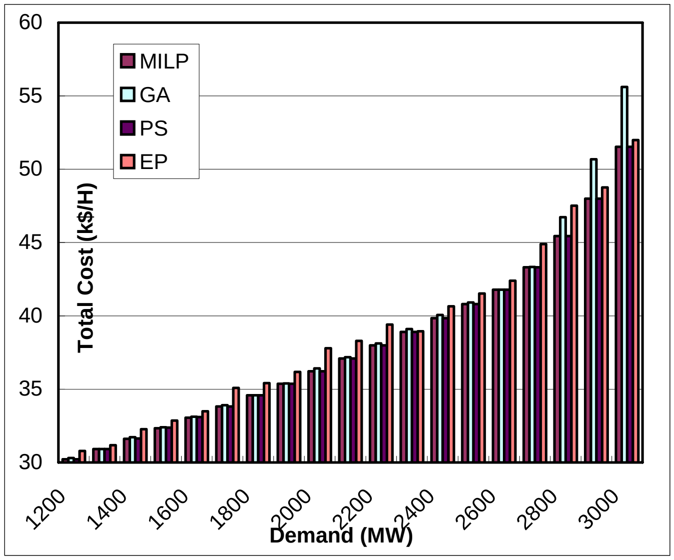

Figure 32: Comparison of Four Algorithms

5.3 Bidding Strategies by Artificial Life Techniques

As discussed in sections 2.1&2.2, computational approaches using autonomous intelligent agents are another way to study electricity market interactions. Sheble [13] developed a single population GA to evolve agents' bidding strategies for a multi-round auction market. A single population evolutionary programming bidding strategy is discussed in [32]. For a market with heterogeneous participants whose strategy spaces are different, co-evolutionary is more appropriate, in which each agent evolves its own population of bidding strategies. Tully used a co-evolutionary GA to investigate the complex market-based unit commitment problem [70]. Thai proposed a co-evolutionary GA to mimic agents' bidding behavior, and simulation results showed participants can improve their trading profits by the learning process [71]. This research will utilize the idea of co-evolutionary to develop EP and PS as well as GA to evolve GENCOs bidding strategies. There are various ways to design the evolutionary operations for the evolutionary process; however, simplicity and ease of implementation [71] are the criteria we use for the design of these learning algorithms.

The most critical problem of co-evolutionary process is fitness evaluation for each individual. For simplicity and low computational burden, “all against the best” [72] is chosen. In this framework, the fitness of an individual from an agent’s population of strategies is evaluated by the trading profit from the simulated competition between it and the best strategies of other agents’ populations. After evaluating a population, the corresponding fittest bidding strategy is marked the best individual for the subsequent fitness evaluation of the other population [71].

In this section, assuming N is the population size of each player position, n is the index of individual, $n = 1, 2, \ldots, N$; I is the number of player positions, i is the index of GENCO, $i = 1, 2, .., I$; Gen is the number of generations, gen is the index of generations, $gen = 1, 2, \ldots, Gen$.

5.3.1 Genetic Algorithm

Step 1: Initialization.

Initialize each individual in every population as an M-bit binary string. Rescale the string into the interval [α_i, α_i+1], where α_i is the linear coefficient of GENCO i’s cost function. α_i: $/per-unit*H

Step 2: Fitness Evaluation.

Use the idea of “all against the best” to evaluate each individual’s fitness. First, one need randomly assign an individual from population 2 to I, when evaluating the fitness of the individuals in population 1. After evaluating a population, the corresponding fittest bidding strategy is marked the best individual for the subsequent fitness evaluation of the other population.

Step 3: Selection.

Form a roulette wheel for each population according to the fitness ranking of individuals. Select a set of individuals by spinning the wheel for each population.

Step 4: Crossover.

Pick two individuals from the post-selected population to perform one-point crossover.

Step 5: Mutation.

Perform mutation prediction for each player population.

Step 6: Elitism.

Perform elitism for each population, *i.e.* keep the best individual of the previous generation in the current one. Form a new generation.

Step 7: $gen = gen + 1$, if $gen > Gen$ go to step 8; otherwise go to step 2.

Step 8: Termination.

5.3.2 Evolutionary Programming

Step 1: Initialization.

Initialize each individual in every population as a uniform distribution random variable within [0, 1]. Rescale the individual into the interval $[\alpha_i, \alpha_i+1]$, where α_i is the linear coefficient of GENCO *i*'s cost function. α_i: \$/per-unit*H

Step 2: Fitness Evaluation.

Use the idea of "all against the best" to evaluate each individual's fitness. First, one need randomly assign an individual from population 2 to *I*, when evaluating the fitness of the individuals in population 1. After evaluating a population, the corresponding fittest bidding strategy is marked the best individual for the subsequent fitness evaluation of the other population.

Step 3: Creation Offsprings.

Perturb each individual by adding a Gaussian random term with mean zero and standard deviation proportional to the ratio between the best and its own fitness.

Step 4: Comparison.

Construct a competing pool for both parent and offspring generations. Use the idea of "all against the best" to construct the score of each individual.

Step 5: Selection.

Pick the first *N* high score individuals as a new generation.

Step 6: $gen = gen + 1$, if $gen > Gen$ go to step 7; otherwise go to step 2.

Step 7: Termination.

5.3.3 Particle Swarm

Step 1: Initialization of positions.

Initialize each individual in every population as a uniform distribution random variable [0, 1]. Rescale the individual into the interval $[\alpha_i, \alpha_i+1]$, where α_i is the linear coefficient of GENCO *i*'s cost function. α_i: $/per-unit*H

Step 2: Fitness Evaluation.

Use the idea of "all against the best" to evaluate each individual's fitness. First, one need randomly assign an individual from population 2 to *I*, when evaluating the fitness of the individuals in population 1. After evaluating a population, the corresponding fittest bidding strategy is marked the best individual for the subsequent fitness evaluation of the other population.

Step 3: Initialization of Velocity

Initialize each velocity in every population as a uniform distribution random variable [0, 1].

Step 4: Initialization of Best values

Record *Pbest* and *Gbest* for each initial population established in step 1.

Step 5: Update Velocity.

Update velocity according to particle swarm dynamics equation (13).

Step 6: Update Position.

Update position according to particle swarm dynamics equation (14).

Step 7: Fitness Re-Evaluation.

Use the idea of "all against the best" to re-evaluate each individual's fitness in the new generation.

Step 8: Update Best Value.

Update *Pbest* and *Gbest* for each population in the new generation.

Step 9: $gen = gen + 1$, if $gen > Gen$ go to step 10; otherwise go to step 5.

Step 10: Termination.

5.3.4 An Example by Particle Swarm

A two-bus example [43] is used to check the feasibility of Particle Swarm to evolve optimal bidding strategy. The system consists of two generators (at Bus 1&2) and one load (at Bus 2 only).

Figure 33: One Line Diagram of a Two-Bus System [43]

GENCO i's offer curve: $P_i = l_i + 0.02q_i$, cost curve: $f_i = 10x_i + \frac{1}{2}0.02x_i^2$

Demand 2's bid curve: $P_2 = 30 - 0.08Q_2$

This is a basic SFE model. The optimal bid for 1&2 (11.63 \$/MWh) and the optimal profit (270.7 \$/h) are given in reference [43].

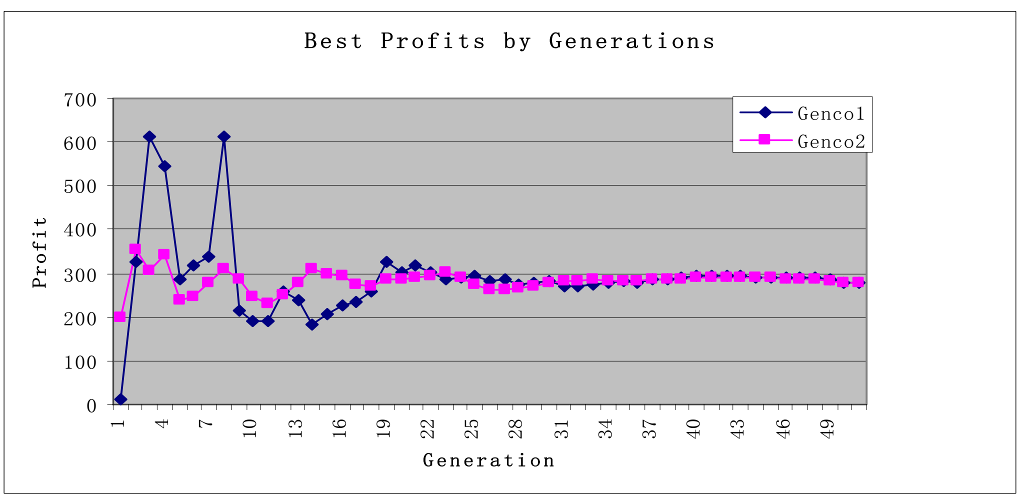

Figure 34: Best Profits per Generation

A dual-population co-evolving Particle Swarm algorithm is implemented based on 5.3.3. Fig. 34 & 35 show how the best profit and bid vary w.r.t generations. It is observed that the algorithm converge to the theoretical optimal solution quickly.

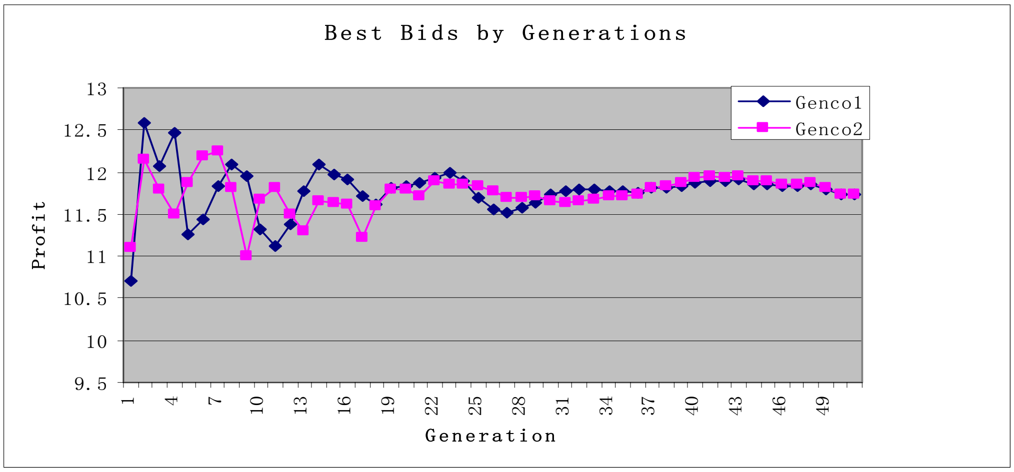

Figure 35: Best Bids per Generation

CHAPTER 6. LINEAR PROGRAMMING

6.1 Introduction

In Chapter 4, a linear supply function equilibrium model is investigated to derive GENCO bidding strategy. The critical assumption is that GENCO submit a linear supply function with 2 strategic variables, slope k and intercept l. This assumption guarantees that theoretical market equilibria can be calculated analytically. If the assumption is relaxed, it is very difficult to find theoretical market equilibria. A lot of works have been done to develop some numerical techniques to solve a nonlinear supply function equilibrium model, generally called Mathematical Programming with Equilibrium Constraints (MPEC). However, MPEC problems are highly non-convex. There is not a commercial software package which can solve MPEC problem robustly so far.

On the other side, almost every electricity market requires participants to submit piecewise staircase energy offer curves (supply curves) that consist of up to 10 - 20 segments. Piecewise staircase curves are defined by multiple variables (both MW breakpoints and prices) which can represent a very non-linear supply function. Linear supply function model does not match the real world very well. Therefore, the previous method cannot be applied to solve the market equilibrium problems in a real world setting.

Given these two concerns, it is very interesting to develop an optimal bidding strategy based on piecewise staircase energy offer curves. The reason why piecewise staircase curves are desirable is that the optimization engine of Economic Dispatch run by ISOs and utilities is based on Linear Programming. Commercial tools of Economic

Dispatch use Linear Programming primarily. Therefore, it is critical to develop an optimal bidding strategy for a GENCO considering the characteristics of LP engine inside ISO. Here a LP based method to derive GENCO bidding strategy where a piecewise staircase curve is assumed.

6.2 Complete Information

In the first part, one assumes that a GENCO has complete information on system conditions (demand, transmission limit, outage schedule etc.) and his rival's strategies (bids/offers).

6.2.1 A low level problem of MPEC – Economic Dispatch

The Economic Dispatch run by ISO can be described in an abstract level by a LP problem with upper bound (LPUB). Security Constraint (e.g. n-1 contingency) can be also incorporated into the model.

Primal Problem:

$$\begin{aligned} &\min\ c^T x \\ &\text{s.t. } Ax = b \rightarrow \lambda \quad &&\text{(1) DC power flow and transmission line constraint} \\ &\qquad 0 \le x \le \overline{x} \rightarrow \mu \quad &&\text{(2) Generator operational constraint} \end{aligned}$$

Where:

c is bids/offers submit by GENCOs

x is decision variables / generator dispatch output at each segment

Considering the dual problem of EDC, it can be described as follows:

Dual Problem:

$$\begin{aligned} \max \quad & b^T \lambda - \overline{x}^T \mu \\ \text{s.t.} \quad & A^T \lambda - \mu \le c \quad (3) \\ & \mu \ge 0 \qquad\qquad (4) \\ & \lambda \text{ is free} \end{aligned}$$

Therefore, the KKT condition for EDC is as follows

$$
\textbf{KKT Conditions:} \quad
\begin{aligned}
&Ax = b \\
&A^T\lambda - \mu \le c \\
&0 \le x \le \overline{x} \\
&\mu \ge 0 \\
&\lambda \text{ is free} \\
&(A^T\lambda - \mu - c)^T x = 0 \quad (5) \\
&(x - \overline{x})^T \mu = 0 \qquad\quad (6)
\end{aligned}
$$

Equation (5) and (6) are called complementary slackness conditions.

6.2.2 A high level problem of MPEC – GENCO profit maximization

$$\max_{c_{ij}} \sum_{i=1}^{\overline{i}} \chi_i \left(\sum_{j=1}^{j_i} x_{ij}\right) - \sum_{i=1}^{\overline{i}} \sum_{j=1}^{j_i} f_{ij} x_{ij}$$

Where:

χ_i: Locational Marginal Price (LMP) for generator i; χ_i is a linear combination of dual variables λ;

f_{ij}: coefficient of piecewise staircase cost curve for generator i, segment j;

This objective function is not linear itself, which consists of both primal and dual variables. The constraints for the high level problem are exactly the same with the KKT conditions of EDC. Obviously the feasible region defined by KKT is not convex since there exist complementary slackness conditions. Generally, the problem is a Mathematical Programming with Equilibrium Constraint (MPEC) or bi-level optimization. It is very challenging to find the global optima of MPEC. The difficulties associated with MPEC are discussed in [76] and [77]. Some numerical algorithms such as PIPA, PSQP, … are proposed in the literature; however, these methods cannot guarantee to find a global optimal solution.

6.2.3 A New Algorithm: Parametric LP and LP

The author proposes a method, which consists of 2 steps: parametric LP and LP. The method uses a famous conclusion in LP: an optimal solution of LP has to be located at an extreme point of feasible region. The global optima will be obtained within finite steps.[75][80][82]

6.2.4 Algorithm Overview

1. Parametric Linear Programming

Parametric Linear Programming is frequently useful to study the behavior of the optimal solution to a LP problem, as the entire objective function is systematically varied, or as the entire requirements vector is systematically varied.

Let us consider the case in which the objective function is varied parametrically.

$$\begin{aligned} &\min\ (c+\alpha\bar{c})^T x \\ &\text{s.t. } Ax = b \rightarrow \lambda \\ &\qquad 0 \le x \le \bar{x} \rightarrow \mu \end{aligned}$$

Where:

c is a bid vector submit by the rivals, with several zero values indicating the GENCO's decision variables; and non-zero for other rivals.

$\overline{c}$ is a bid vector submit by the GENCO, with several non-zero values indicating the GENCO's decision variables; and zero for other rivals.

α is a scalar parameter ($\alpha \geq 0$). α can be explained as a multiplier or strategic variable between bids and costs, which is also a decision variable of the GENCO. If $\overline{c}$ represents the true marginal cost of the GENCO, α represents a bid "mark-up" ($\alpha > 1$) or "mark-down" ($\alpha > 1$).

x is the vector of decision variables / generator dispatch output at each segment.

Assume one has found the optimal solution x^* at $\alpha = \alpha^*$. B is the optimal basis (the vectors that basic variables correspond to), N is the vectors that lower non-basic variables correspond to, and M is the vectors that upper non-basic variables correspond to. Then one can write A matrix and cost vectors c as follows:

$$A = [B, N, M]$$
$$c = [c_B, c_N, c_M]$$

According to the optimal condition of LPUP:

$$\xi_N = c_B B^{-1} N - c_N \leq 0$$
$$\xi_M = c_B B^{-1} M - c_M \geq 0$$

One can know that these two inequalities must hold given $\alpha = \alpha^*$.

$$\xi_N + \alpha^* \overline{\xi}_N = (c_B B^{-1} N - c_N) + \alpha^* (\overline{c}_B B^{-1} N - \overline{c}_N) \leq 0 \qquad (7)$$
$$\xi_M + \alpha^* \overline{\xi}_M = (c_B B^{-1} M - c_M) + \alpha^* (\overline{c}_B B^{-1} M - \overline{c}_M) \geq 0 \qquad (8)$$

In order to calculate the interval of α, within which x^* is the optimal solution, one need to solve (7) and (8) for j = 1, 2, …, n. n is the number of non-basic variables.

Let us consider equation (7) first.

$$\begin{aligned} &\xi_{N,j} \le 0 \ \ if \ \ \bar{\xi}_{N,j} = 0 \\ &\alpha \le -\xi_{N,j} / \bar{\xi}_{N,j} \ \ if \ \ \bar{\xi}_{N,j} > 0 \\ &\alpha \ge -\xi_{N,j} / \bar{\xi}_{N,j} \ \ if \ \ \bar{\xi}_{N,j} < 0 \end{aligned}$$

Let us define

$$\begin{aligned} &\bar{\alpha}_N = \min\{-\xi_{N,j} / \bar{\xi}_{N,j} \ \ | \bar{\xi}_{N,j} > 0\} \\ &\underline{\alpha}_N = \max\{-\xi_{N,j} / \bar{\xi}_{N,j} \ \ | \bar{\xi}_{N,j} < 0\} \end{aligned}$$

Then, let us consider equation (8).

$$\begin{aligned} &\xi_{M,j} \ge 0 \ \ if \ \ \bar{\xi}_{M,j} = 0 \\ &\alpha \ge -\xi_{M,j} / \bar{\xi}_{M,j} \ \ if \ \ \bar{\xi}_{M,j} > 0 \\ &\alpha \le -\xi_{M,j} / \bar{\xi}_{M,j} \ \ if \ \ \bar{\xi}_{M,j} < 0 \end{aligned}$$

Let us define

$$\begin{aligned} &\bar{\alpha}_M = \min\{-\xi_{M,j} / \bar{\xi}_{M,j} \ \ | \bar{\xi}_{M,j} < 0\} \\ &\underline{\alpha}_M = \max\{-\xi_{M,j} / \bar{\xi}_{M,j} \ \ | \bar{\xi}_{M,j} > 0\} \end{aligned}$$

Here:

$$\left(\begin{aligned} &\min \varnothing = +\infty \\ &\max \varnothing = -\infty \end{aligned} \right)$$

Then one can define $\begin{aligned} &\bar{\alpha}_B = \min\{\bar{\alpha}_M, \bar{\alpha}_N\} \\ &\underline{\alpha}_B = \max\{\underline{\alpha}_M, \underline{\alpha}_N\} \end{aligned}$, therefore for $\alpha \in [\underline{\alpha}_B, \bar{\alpha}_B]$, the optimal solution x^* does not change. The interval of $\alpha \in [\underline{\alpha}_B, \bar{\alpha}_B]$ is called the characteristic interval of basis B.

If α exceeds $\bar{\alpha}_B$, the current solution is no longer optimal. In this case, one would find the new optimal solution resulting from letting α be slightly larger than $\bar{\alpha}_B$, and then finding the new upper limit on α for which this solution remains optimal. One could repeat this process for whatever range of value for α were of interest.

2. Linear Programming.

Assuming the current solution x^* (vertex) is found, the GENCO is trying to maximize its own profit by changing its bids.

Then in this step, the GENCO profit maximization becomes

$$\begin{aligned}
&\max_{\tilde{c}_{ij}} \sum_{i=1}^{\bar{i}} \chi_i[\lambda](\sum_{j=1}^{j_i} x_{ij}) - \sum_{i=1}^{\bar{i}} \sum_{j=1}^{j_i} f_{ij} x_{ij} \\
&\text{s.t.} \\
&\quad A^T \lambda - \mu \le \tilde{c} \\
&\quad \mu \ge 0 \\
&\quad (A^T \lambda - \mu - \tilde{c})^T x = 0 \\
&\quad (x - \bar{x})^T \mu = 0 \\
&\quad \lambda \text{ is free}
\end{aligned} \qquad (9)$$

Where:

$\chi_i[\lambda]$: Locational Marginal Price (LMP) for generator *i*; $\chi_i[\lambda]$ is a linear combination of dual variables λ;

The decision variables include $\tilde{c}$, λ, and μ. $\tilde{c}$ is equal to $c + \alpha\bar{c}$.

Initially, one may think this problem is hard to solve since complementary slackness conditions still are there. However, given this assumption “the optimal solution (vertex) does not change”, it is true that decision variable x is already known ($x = x^*$)! So complementary slackness conditions become linear. This is really a linear programming problem. After solving this, one will get the optimal bidding at this dispatch optimal solution (vertex).

3. Repeat LP in step 2 and calculate the optimal bids at a new solution of parametric LP.

4. This procedure will be finite since a polyhedron (defined by linear constraints of the low level problem/EDC) has a limit number of vertex.

The optimal bidding of a GENCO will be obtained by solving multiple LP problems and choosing the one associated with the maximal profit. Considering LP solver is very fast now, it will not be a big issue.

5. An extended application for this method is to incorporate incomplete information.

6.3 Incomplete Information

In the second part, one assumes that a GENCO has incomplete information on system conditions (demand, transmission limit, outage schedule etc.) and his rival's strategies (bids/offers).

6.3.1 Incomplete Information and Decision Analysis

Based on the classical Decision Analysis method, the incomplete information is represented by a set of scenarios $\omega \in \Omega$, Ω is the space of scenarios [64]. It is assumed that each scenario ω is independent. Fig. 36 shows a decision tree with each scenario.

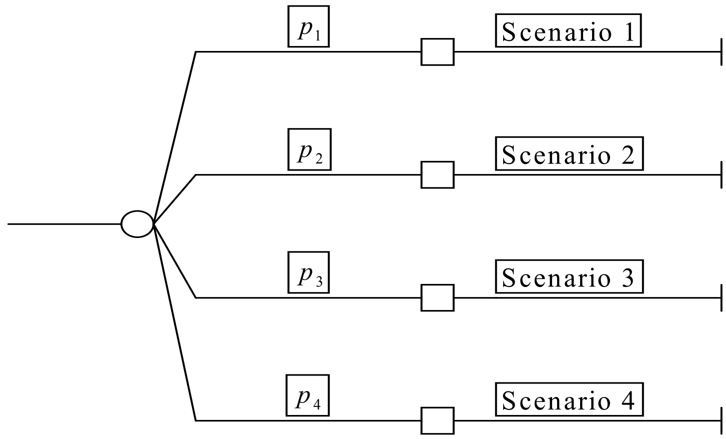

Figure 36: Decision Tree with Each Scenario

6.3.2 GENCO profit maximization with incomplete Information

This objective function with incomplete information changes to maximize the expected profit over the scenarios space Ω.

The constraint set is the intersection of each constraint set associated with each scenario.

Therefore, the GENCO profit maximization problem with incomplete information can be formulized as follows:

$$\max_{\tilde{c}_{ij}^{\omega}} \; E_{\omega}\left[\sum_{i=1}^{\bar{i}} \chi_i[\lambda^{\omega}](\sum_{j=1}^{j_i} x_{ij}^{\omega}) - \sum_{i=1}^{\bar{i}} \sum_{j=1}^{j_i} f_{ij} x_{ij}^{\omega}\right]$$

s.t.

$$\begin{aligned}
& A^{\omega T}\lambda^{\omega} - \mu^{\omega} \le \tilde{c}^{\omega} \\
& \mu^{\omega} \ge 0 \\
& (A^{\omega T}\lambda^{\omega} - \mu^{\omega} - \tilde{c}^{\omega})^{T} x^{\omega} = 0 \\
& (x^{\omega} - \bar{x}^{\omega})^{T} \mu^{\omega} = 0 \\
& \lambda^{\omega} \text{ is free} \\
& \omega \in \Omega
\end{aligned} \qquad (10)$$

Where:

$\tilde{c}_{ij}^{\omega}$ is bids submitted by the GENCO which is under study.

The decision variables include $\tilde{c}^{\omega}$, λ^{ω}, and μ^{ω}. $\tilde{c}^{\omega}$ is equal to $c^{\omega} + \alpha\bar{c}$.

6.3.3 A New Algorithm: Scenario Analysis, Parametric LP, and LP

The author proposes a scenario-based method. Each scenario consists of 2 steps: parametric LP and LP. The global optima will be obtained within finite steps. The method uses a famous conclusion in LP: an optimal solution of LP has to be located at an extreme point of feasible region. The global optima will be obtained within finite steps.

6.3.4 Algorithm Overview

1. Parametric Linear Programming.

For each scenario ω, it is complete information. Parametric Linear Programming is carried on to look for characteristic interval and the associated optimal solution as before. With α increasing, the union set of each solution is a vertex of the whole problem.

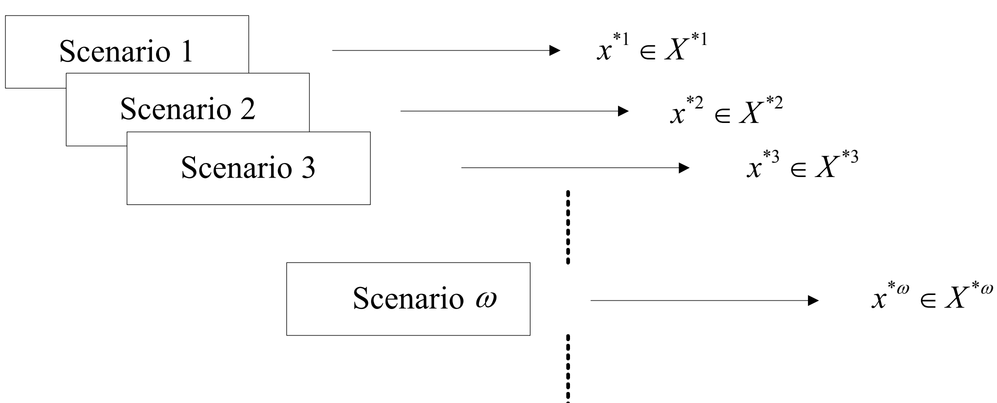

Figure 37: The Vertex with Each Scenario

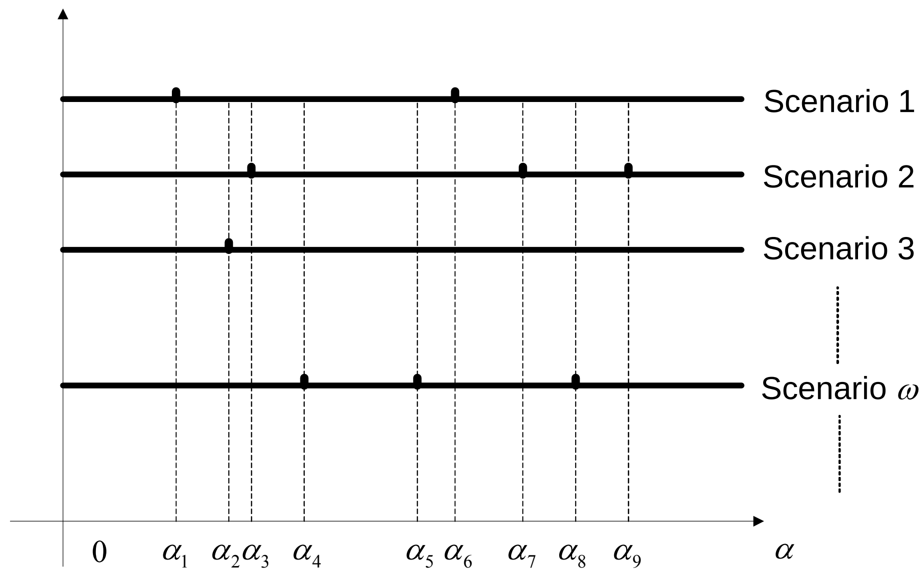

Figure 38: The Combination of Vertex

2. Linear Programming.

Give each solution from PLP $x^* \in X^* = X^{*1} \cup X^{*2} \cup \ldots \cup X^{*\omega} \cup \ldots$, one can solve problem (10) as a linear programming. Since the number of solution of PLP is finite, the global optima will be obtained within finite steps.

6.4 Numerical Examples

6.4.1 4-Bus System

In Section 6.4.1, the proposed method is applied to a 4-bus illustrative system. The optimal bid is simulated case by case.

The one-line diagram of 4-bus system is shown in Fig. 39. The load data, branch admittance, and flow direction are marked in Fig. 39. Table 11 & 12 list generators cost data and business associates cross-reference.

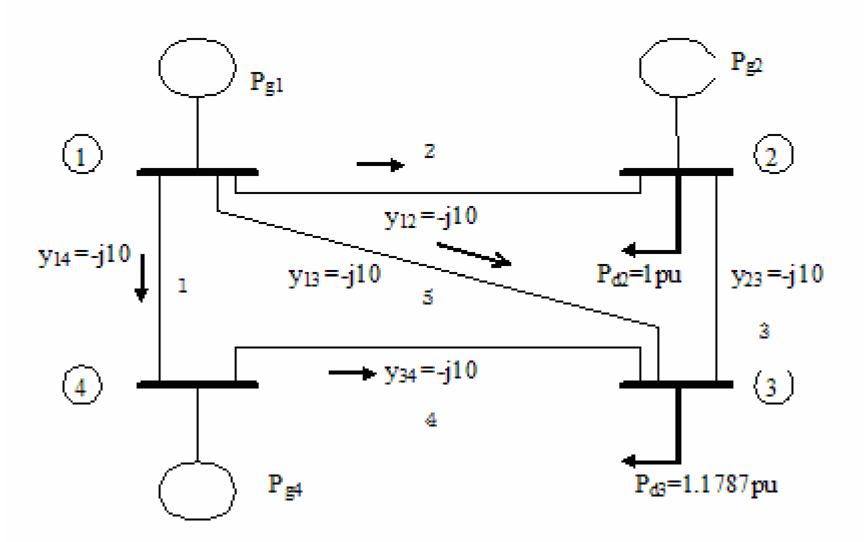

Figure 39: One Line Diagram for 4-Bus System [4]

Table 11: Slopes of Piecewise Linear Cost Curves for 4-Bus System

Company	**Unit *i***	$f_{i,1}$	$f_{i,2}$	$f_{i,3}$
A	1	12.46	13.07	13.58
	2	11.29	12.11	12.82
B	4	11.83	12.54	13.20

Table 12: Break Points of Piecewise Linear Cost Curves for 4-Bus System

Company	**Unit *i***	**Min Gen Cost ($)**	**BP_1 (MW)**	**BP_2 (MW)**	**BP_3 (MW)**	**BP_4 (MW)**
A	1	809.9	50.0	100	160	200
	2	600	37.5	70	130	150
B	4	742.5	45.0	90	140	180

6.4.1.1 Complete Information

Case 1 Base Case

Table 13: Demand for 4-Bus System [4]

Load *j*	**MW**
P_{d2}	100
P_{d3}	117.87

Case 1 is considered as a “Base Case”. Table 13 lists load data of the system. Table 14 defines transmission capacity. From Case 1 to Case 4, it is assumed that transmission line has infinite capacity.

Table 14: Transmission Capacity for Base Case in 4-Bus System [4]

Transmission Line	**Capacity**
1	Inf
2	Inf
3	Inf
4	Inf
5	Inf

Case 1 assumes Company B will bid as his marginal cost (shown in Table 15). This assumption is applied in Case 1 – Case 3 & Case 5. Company A is looking for his optimal bidding strategy based on the strategic variable α.

Table 15: Bids for 4-Bus System

Company	**Unit *i***	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$
A	1	1.1α	1.2α	1.3α
	2	1.05α	1.15α	1.25α
B	4	11.83	12.54	13.20

After running the proposed algorithm, the critical points by PLP are shown in Table 16. Fig. 40 and Table 17 show that how the maximal profit of Company A changes with respect to his bid. The maximal profit is obtained by bidding at $\alpha = 11.4$.

According to Fig. 40, the maximal profit of Company A is a piecewise linear function of his bid. There are several jumps in the function because all of bid curves are piecewise staircase. Obviously the profit function is highly non-convex, which is understandable since the GENCO profit maximization is essentially a MPEC problem

Table 16: Critical Points for Base Case in 4-Bus System

Critical Point	0	10.287	10.755	11.4	11.943	Inf

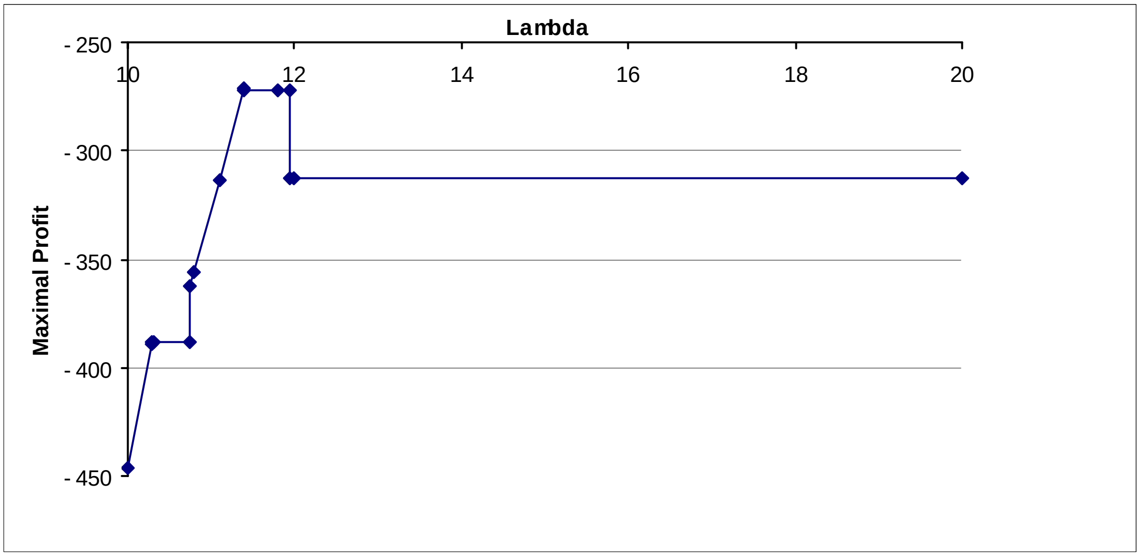

Figure 40: Company A Profit for Base Case in 4-Bus System

Table 17: Profit for Base Case in 4-Bus System

Alpha	**Profit**
5	-1440.55
10	-446.551
10.287	-389.504
10.287	-388.7
10.3	-388.7
10.7545	-388.7
10.7545	-362.136
10.8	-355.765
11.1	-313.568
11.4	-271.37
11.4	-272
11.8	-272
11.9429	-272
11.9429	-312.625
12	-312.625
20	-312.625

Table 18 lists generation dispatch MW of each segment, which is corresponding to critical intervals.

Table 18: Dispatch MW for Base Case in 4-Bus System

Alpha / MW	1	10.7	10.8	11.5	12
MW_{11}	0.5	0.5	0.0787	0	0
MW_{12}	0	0	0	0	0
MW_{13}	0	0	0	0	0
MW_{21}	0.325	0.325	0.325	0.325	0
MW_{22}	0.0287	0	0	0	0
MW_{23}	0	0	0	0	0
MW_{41}	0	0.0287	0.45	0.45	0.45
MW_{42}	0	0	0	0.0787	0.4037
MW_{43}	0	0	0	0	0

Case 2 High Demand

In Case 2, it is assumed that each load increases by 100%. Table 19 lists load data of the system. Table 20 shows the critical points by PLP.

Table 19: High Demand (200%) for 4-Bus System

Load *j*	**MW**
P_{d2}	200
P_{d3}	235.74

Table 20: Critical Points for Case 2 in 4-Bus System

Critical Point	9.1	9.6462	10.032	10.56	11	83.3333	86.9565	90.9091	95.2381	Inf

Fig. 41 and Table 21 show maximal profit with respect to bid. And Table 22 lists generation dispatch MW of each segment, which is corresponding to critical intervals.

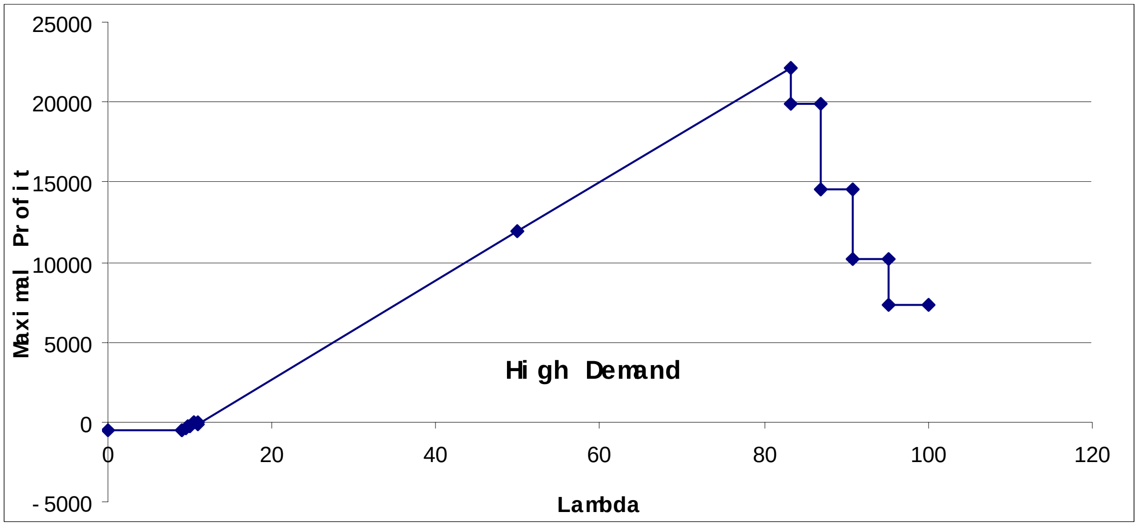

Figure 41: Company A Profit for Case 2 in 4-Bus System

Table 21: Profit for Case 2 in 4-Bus System

Alpha	Profit	Alpha	Profit
0	-569.7	11	-87.0538
9.1	-569.7	50	11881.58
9.1	-562.245	83.3333	22111.17
9.5	-382.46	83.3333	19873.6
9.6461	-316.79	86.9565	19873.6
9.6462	-279.6	86.9565	14600.2
10.032	-279.6	90.909	14600.2
10.032	-275.607	90.9091	10223.2
10.56	-80.4188	95.238	10223.2
10.56	-82.6	95.2381	7340.125
11	-82.6	100	7340.125

Table 22: Dispatch MW for Case 2 in 4-Bus System

Alpha / MW	8	9.5	10	10.5	10.9	50	85	88	95	200
MW_{11}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0	0
MW_{12}	0.6	0.6	0.6	0.6	0.6	0.2574	0	0	0	0
MW_{13}	0.4	0.3574	0	0	0	0	0	0	0	0
MW_{21}	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0
MW_{22}	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0	0	0
MW_{23}	0.2	0.2	0.2	0.0574	0	0	0	0	0	0
MW_{41}	0.4074	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
MW_{42}	0	0	0.3574	0.5	0.5	0.5	0.5	0.5	0.5	0.5
MW_{43}	0	0	0	0	0.0574	0.4	0.4	0.4	0.4	0.4

Case 3 Low Demand

In Case 3, it is assumed that each load decreases by 20% (Shown in Table 23).

Table 23: High Demand (80%) for 4-Bus System

Load *j*	MW
P_{d2}	80
P_{d3}	94.30

Table 24: Critical Points for Case 3 in 4-Bus System

Critical Point	0	10.7545	11.2667	Inf

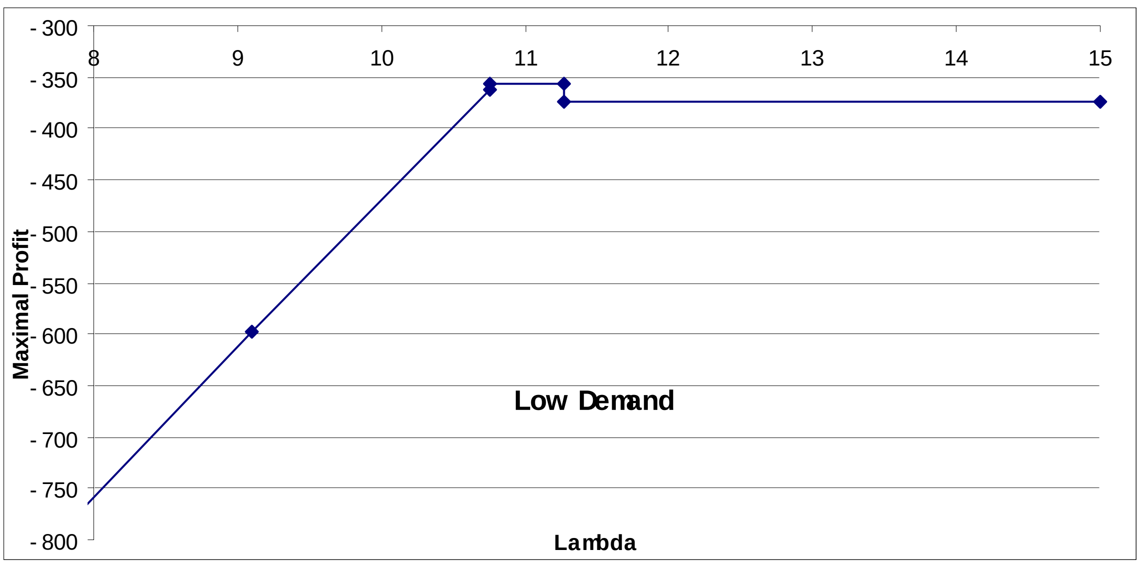

Figure 42: Company A Profit for Case 3 in 4-Bus System

Table 25: Profit for Case 3 in 4-Bus System

Alpha	Profit
0	-1930.7
9.1	-598.385
10.7545	-363.065
10.7545	-357.2
11.2667	-357.2
11.2667	-374.75
15	-374.75

Table 26: Dispatch MW for Case 3 in 4-Bus System

Alpha / MW	1	11	50
MW_{11}	0.093	0	0
MW_{12}	0	0	0
MW_{13}	0	0	0
MW_{21}	0.325	0.325	0
MW_{22}	0	0	0
MW_{23}	0	0	0
MW_{41}	0	0.093	0.418
MW_{42}	0	0	0
MW_{43}	0	0	0

Case 4 High Bid

Case 4 assumes Company B will bid as 20% higher than his marginal cost (Shown in Table 27).

Table 27: High Bids for 4-Bus System

Company	Unit i	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$
A	1	1.1α	1.2α	1.3α
	2	1.05α	1.15α	1.25α
B	4	14.20	15.05	15.84

Table 28: Critical Points for Case 4 in 4-Bus System

Critical Point	0	12.34	12.90	13.68	14.33	Inf

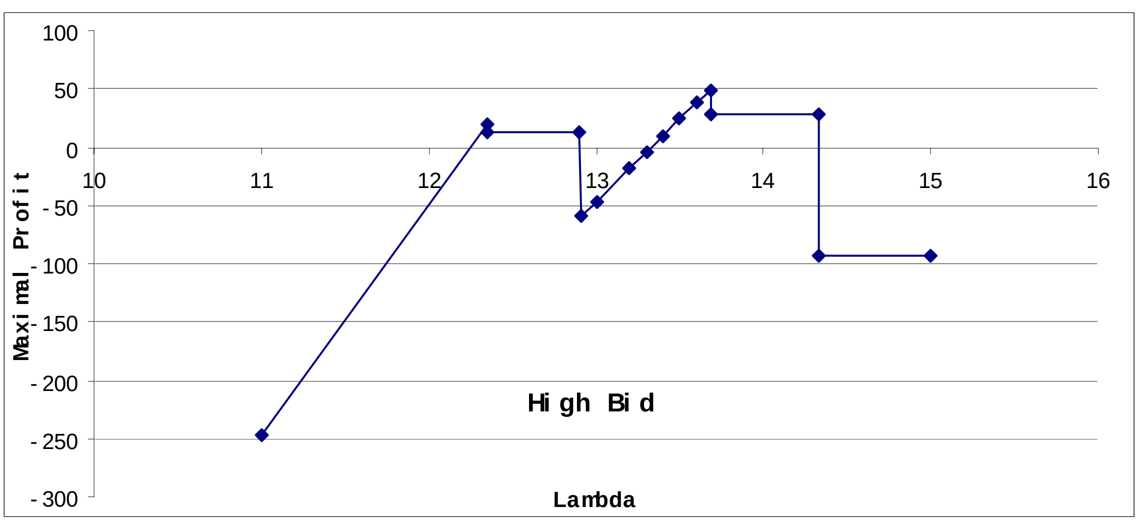

Figure 43: Company A Profit for Case 4 in 4-Bus System

Table 29: Profit for Case 4 in 4-Bus System

Alpha	Profit	Alpha	Profit
11	-247.7502	13.4	9.9436
12.3443	19.49731215	13.5	24.0093
12.3443	13.52	13.6	38.075
12.9005	13.52	13.68	49.32756
12.91	-58.97833	13.68	28.96
13	-46.3192	14.3314	28.96
13.2	-18.1878	14.3315	-93.175
13.3	-4.1221	15	-93.175

Table 30: Dispatch MW for Case 4 in 4-Bus System

Alpha / **MW**	**5**	**12.8**	**13.5**	**14**	**15**
MW_{11}	0.5	0.5	0.0787	0	0
MW_{12}	0	0	0	0	0
MW_{13}	0	0	0	0	0
MW_{21}	0.325	0.325	0.325	0.325	0
MW_{22}	0.0287	0	0	0	0
MW_{23}	0	0	0	0	0
MW_{41}	0	0.0287	0.45	0.45	0.45
MW_{42}	0	0	0	0.0787	0.4037
MW_{43}	0	0	0	0	0

Case 5 Transmission Congestion

Case 5 assumes that transmission line 3 has a capacity of 10 MW (Shown in Table 31). Transmission congestion may happen due to the small limit. In this dissertation, transmission limits only refer to thermal limits. Voltage and stability limits [81] may be incorporated, but it is beyond the scope of this research.

Table 31: Transmission Capacity for Case 5 in 4-Bus System

Transmission Line	**Capacity**
1	Inf
2	Inf
3	10
4	Inf
5	Inf

Table 32: Critical Points for Case 5 in 4-Bus System

Critical Point	0	9.464	10.594	10.7545	11.4	11.9429	95.2381	Inf

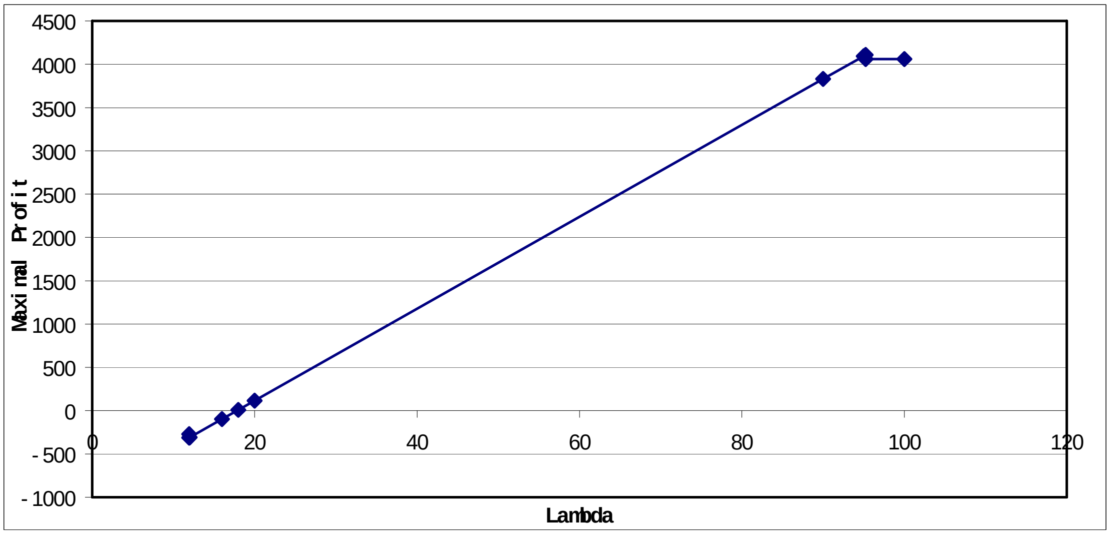

Figure 44: Company A Profit for Case 5 in 4-Bus System

Table 33: Profit for Case 5 in 4-Bus System

Alpha	Profit	Alpha	Profit
0	-2449.6107	11.4	-271.3704
9.464	-575.923248	11.4	-272
9.464	-562.468984	11.9429	-272
10.594	-455.055139	11.9429	-311.9602261
10.6	-439.004	95.2381	4107.3913
10.7545	-377.4600065	95.2381	4060.375
10.7546	-362.1504278	100	4060.375

Table 34: Dispatch MW for Case 5 in 4-Bus System

Alpha / **MW**	**5**	**10**	**10.7**	**11**	**11.5**	**20**	**100**
MW_{11}	0.5	0.5	0.3213	0.0787	0	0	0
MW_{12}	0.0978	0	0	0	0	0	0
MW_{13}	0	0	0	0	0	0	0
MW_{21}	0.2559	0.2803	0.325	0.325	0.325	0.0053	0
MW_{22}	0	0	0	0	0	0	0
MW_{23}	0	0	0	0	0	0	0
MW_{41}	0	0.0734	0.2074	0.45	0.45	0.45	0.45
MW_{42}	0	0	0	0	0.0787	0.3984	0.3984
MW_{43}	0	0	0	0	0	0	0

6.4.1.2 Incomplete Information

Table 35: Probability of Each Scenario

Scenarios	Probability
Base Case	0.45
High Bid	0.05
Low Demand	0.45
High Demand	0.05

Table 36: Profit for Incomplete Information Case in 4-Bus System

Alpha	Profit	Alpha	Profit
9.1	-610.494	11.9429	-283.943
9.6462	-506.738	12.3443	-292.069
10.032	-443.697	12.9005	-283.833
10.287	-397.119	13.68	-270.082
10.56	-371.525	14.3314	-260.993
10.7545	-357.252	83.3333	791.5809
11	-324.915	86.9565	679.7025
11.2667	-309.188	90.9091	197.1825
11.4	-297.38	95.2381	53.02875

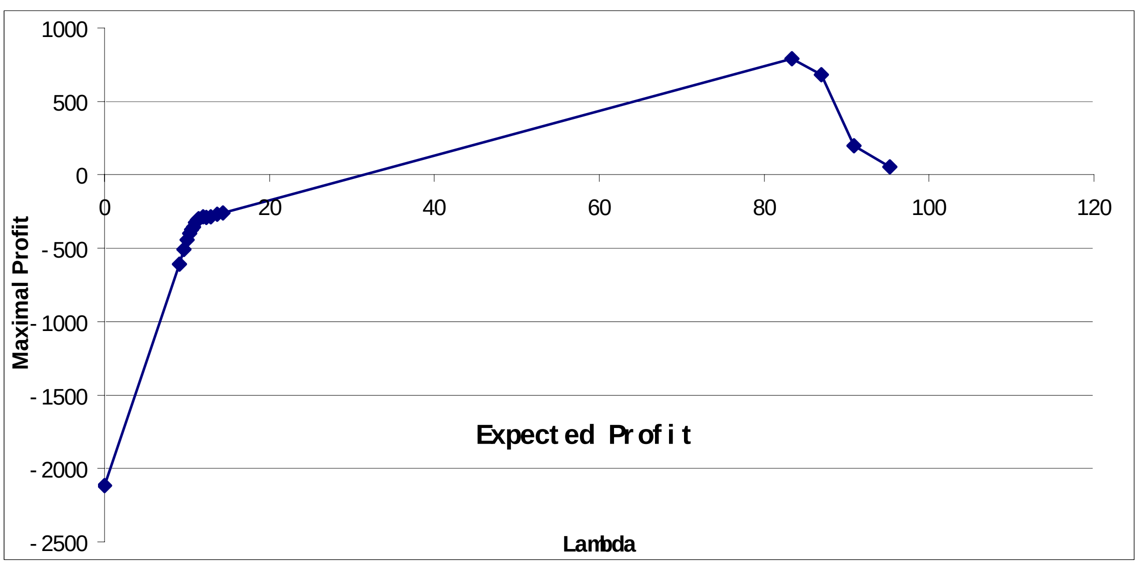

Figure 45: Company A Profit for Incomplet Information in 4-Bus System

6.4.2 RTS96 System

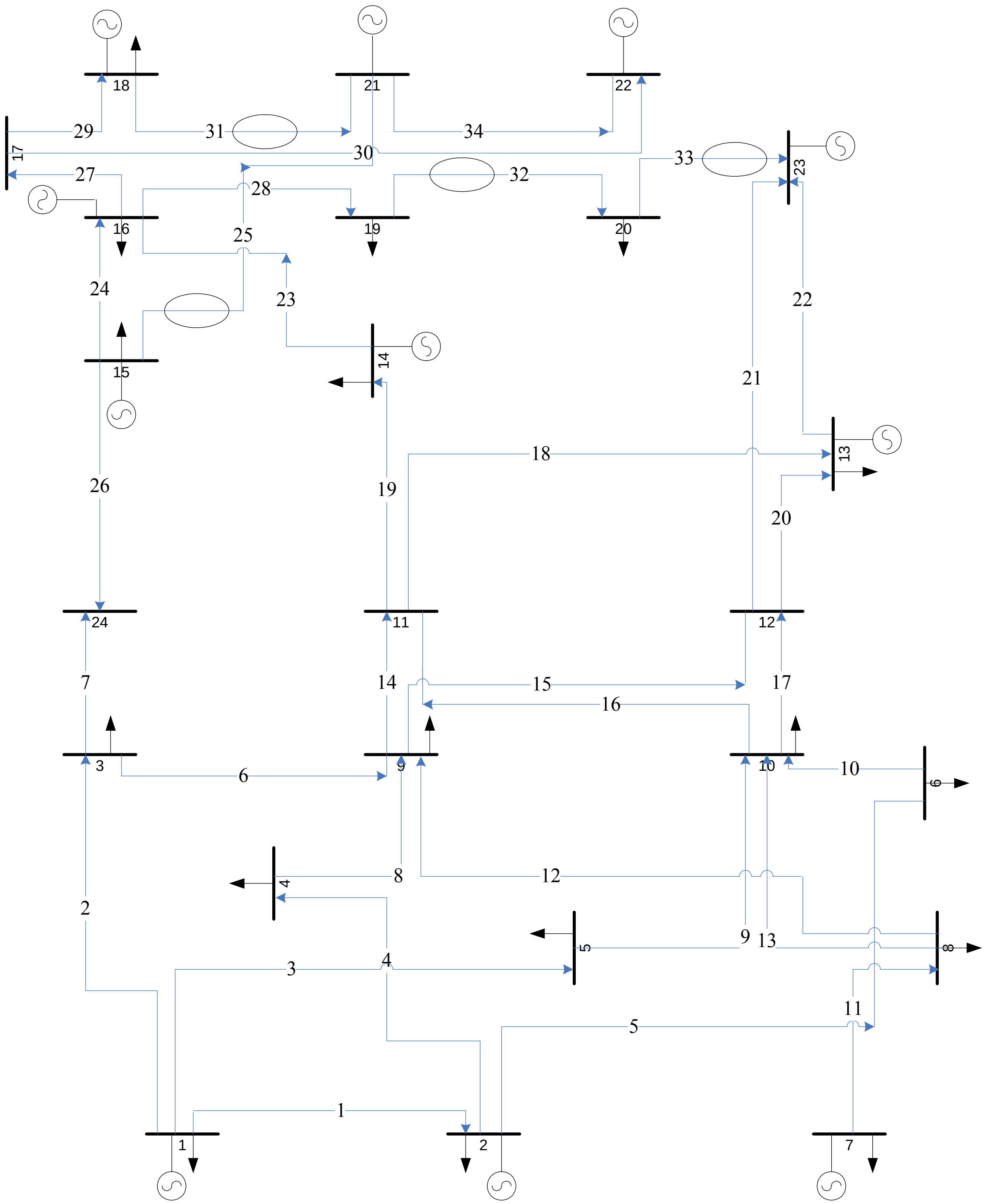

Figure 46: One Line Diagram for RTS96 System [73]

In Section 6.4.2, the proposed method is applied to RTS96 test system. RTS96 system is a standard benchmark example in bulk power system reliability evaluation studies [73]. The topology for RTS96 is shown in Fig. 46 and is considered as ”One Area”. A ”Two Area” or ”Three Area” system can be developed by linking various single RTS96 areas. This research will focus on One Area system. The system includes 24 buses, 34 transmission lines (4 double-lines), 11 units, and 17 loads. In order to simulate optimal bidding strategy, the whole system is divided into 3 companies, which is shown in Fig. 47 and Table 37. Company A holds 54.2% of total capacity, Company B holds 16.6%, and Company C holds 29.2%. Company C is the one under study, A&B are considered as competitors.

Table 38 shows the generating unit heat rates. Table 39 shows the assumed load for each bus. Table 40 shows fuel cost information from Energy Information Administration website. Table 41 shows the transmission branch data. All *pu* quantities are on 100 MVA base.

Table 37: Business Associations and Generation Units

Company	**Capacity MW**	**Bus ID**	**Unit Type**
A	1052	2	U76
		7	U100
		18	U400
		21	U400
		22	U76
B	322	14	U155
		15	U12
		16	U155
C	567	1	U20
		13	U197
		23	U350

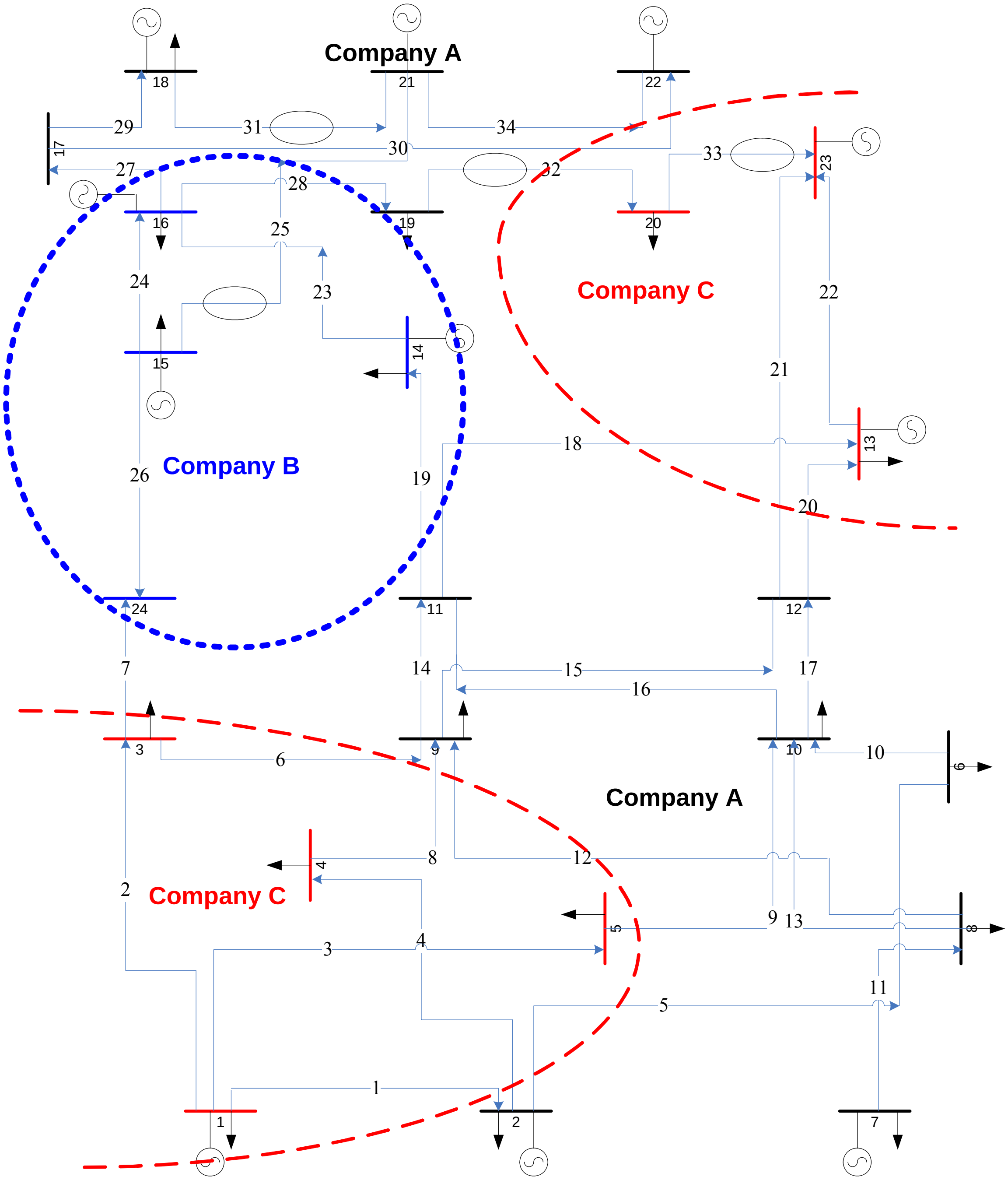

Figure 47: Bussiness Regions for RTS96 System

Table 38: Generator Heat Rate and Incremental Heat Rate [73]

Size MW	Type	Fuel	Output %	MW	Net Plant Heat Rate (MBTU/MWH)	Incremental Heat Rate Calculated by Continuous Function (MBTU/MWH)
12	Fossil Steam	#6 Oil	20	2.40	16.017	10.179
			50	6.00	12.500	10.330
			80	9.60	11.900	11.668
			100	12.00	12.000	13.219
20	Combustion Turbine	Natural Gas	79	15.80	15.063	9.859
			80	16.00	15.000	10.139
			99	19.80	14.500	14.272
			100	20.00	14.499	14.427
50	Hydro		100	50.00	Not Applicable	
76	Fossil Steam	Coal	20	15.20	17.107	9.548
			50	38.00	12.637	9.966
			80	60.80	11.900	11.576
			100	76.00	12.000	13.311
100	Fossil Steam	#6 Oil	25	25.00	12.999	8.089
			50	50.00	10.700	8.708
			80	80.00	10.087	9.420
			100	100.00	10.000	9.877
155	Fossil Steam	Coal	35	54.25	11.244	8.265
			60	93.00	10.053	8.541
			80	124.00	9.718	8.900
			100	155.00	9.600	9.381
197	Fossil Steam	#6 Oil	35	68.95	10.750	8.348
			60	118.20	9.850	8.833
			80	157.60	9.644	9.225
			100	197.00	9.600	9.620
350	Fossil Steam	Coal	40	140.00	10.200	8.402
			65	227.50	9.600	8.896
			80	280.00	9.500	9.244
			100	350.00	9.500	9.768
400	Fossil Steam	Coal	25	100.00	12.751	8.848
			50	200.00	10.825	8.965
			80	320.00	10.170	9.210
			100	400.00	10.000	9.438

Table 39: Bus Load Data [73]

Bus ID	Bus Load
	% Of System Load
1	3.8
2	3.4
3	6.3
4	2.6
5	2.5
6	4.8
7	4.4
8	6.0
9	6.1
10	6.8
13	9.3
14	6.8
15	11.1
16	3.5
18	11.7
19	6.4
20	4.5
	Total 100.0

Table 40: Fuel Cost Data [78]

Fuel Type	Cost ($/MBTU)
Oil	6.44
Coal	1.54
Natural Gas	8.21

Case 1 Base Case

Case 1 is considered as a "Base Case". The system demand is 1000 MW (Table 42). It is assumed that all competitors bid as their marginal costs. Players can estimate competitors' marginal costs based on generating technology, fuel costs, and historic data published on OASIS system. Company C bids on his marginal cost as well. α is defined as a strategic variable or bid mark-up. Company C's bidding pattern is shown in Table 43. When α = 12.94, Company A bids as his marginal cost.

Table 41: Transmission Line Data [73]

Con = Continuous rating
LTE = Long-time emergency rating (24 hour)
STE = Short-time emergency rating (15 minutes)

ID #	From Bus	To Bus	L Miles	R (pu)	X (pu)	B (pu)	Con MVA	LTW MVA	STE MVA
A1	101	102	3	0.003	0.014	0.461	175	193	200
A2	101	103	55	0.055	0.211	0.057	175	208	220
A3	101	105	22	0.022	0.085	0.023	175	208	220
A4	102	104	33	0.033	0.127	0.034	175	208	220
A5	102	106	50	0.050	0.192	0.052	175	208	220
A6	103	109	31	0.031	0.119	0.032	175	208	220
A7	103	124	0	0.002	0.084	0	400	510	600
A8	104	109	27	0.027	0.104	0.028	175	208	220
A9	105	110	23	0.023	0.088	0.024	175	208	220
A10	106	110	16	0.014	0.061	2.459	175	193	200
A11	107	108	16	0.016	0.061	0.017	175	208	220
A12-1	108	109	43	0.043	0.165	0.045	175	208	220
A13-1	108	110	43	0.043	0.165	0.045	175	208	220
A14	109	111	0	0.002	0.084	0	400	510	600
A15	109	112	0	0.002	0.084	0	400	510	600
A16	110	111	0	0.002	0.084	0	400	510	600
A17	110	112	0	0.002	0.084	0	400	510	600
A18	111	113	33	0.006	0.048	0.100	500	600	625
A19	111	114	29	0.005	0.042	0.088	500	600	625
A20	112	113	33	0.006	0.048	0.100	500	600	625
A21	112	123	67	0.012	0.097	0.203	500	600	625
A22	113	123	60	0.011	0.087	0.182	500	600	625
A23	114	116	27	0.005	0.059	0.082	500	600	625
A24	115	116	12	0.002	0.017	0.036	500	600	625
A25-1	115	121	34	0.006	0.049	0.103	500	600	625
A25-2	115	121	34	0.006	0.049	0.103	500	600	625
A26	115	124	36	0.007	0.052	0.109	500	600	625
A27	116	117	18	0.003	0.026	0.055	500	600	625
A28	116	119	16	0.003	0.023	0.049	500	600	625
A29	117	118	10	0.002	0.014	0.030	500	600	625
A30	117	122	73	0.014	0.105	0.221	500	600	625
A31-1	118	121	18	0.003	0.026	0.055	500	600	625
A31-2	118	121	18	0.003	0.026	0.055	500	600	625
A32-1	119	120	27.5	0.005	0.040	0.083	500	600	625
A32-2	119	120	27.5	0.005	0.040	0.083	500	600	625
A33-1	120	123	15	0.003	0.022	0.046	500	600	625
A33-2	120	123	15	0.003	0.022	0.046	500	600	625
A34	121	122	47	0.009	0.068	0.142	500	600	625

Table 42: System Demand for Base Case

Total System Demand (MW)	1000

Table 43: Bids of Piecewise Linear Cost Curves

Company	Unit i	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$
C	1	6.26α	6.43α	9.06α
	13	4.15α	4.40α	4.59α
	23	1.00α	1.06α	1.10α

Fig. 48 and Table 44 show that how the maximal profit of Company C changes with respect to his bid. The maximal profit is obtained by bidding between $\alpha = 12.52$ and 13.009.

According to Fig. 48, the maximal profit of Company C is a piecewise linear function of his bid, which is consistent with the previous 4-bus example.

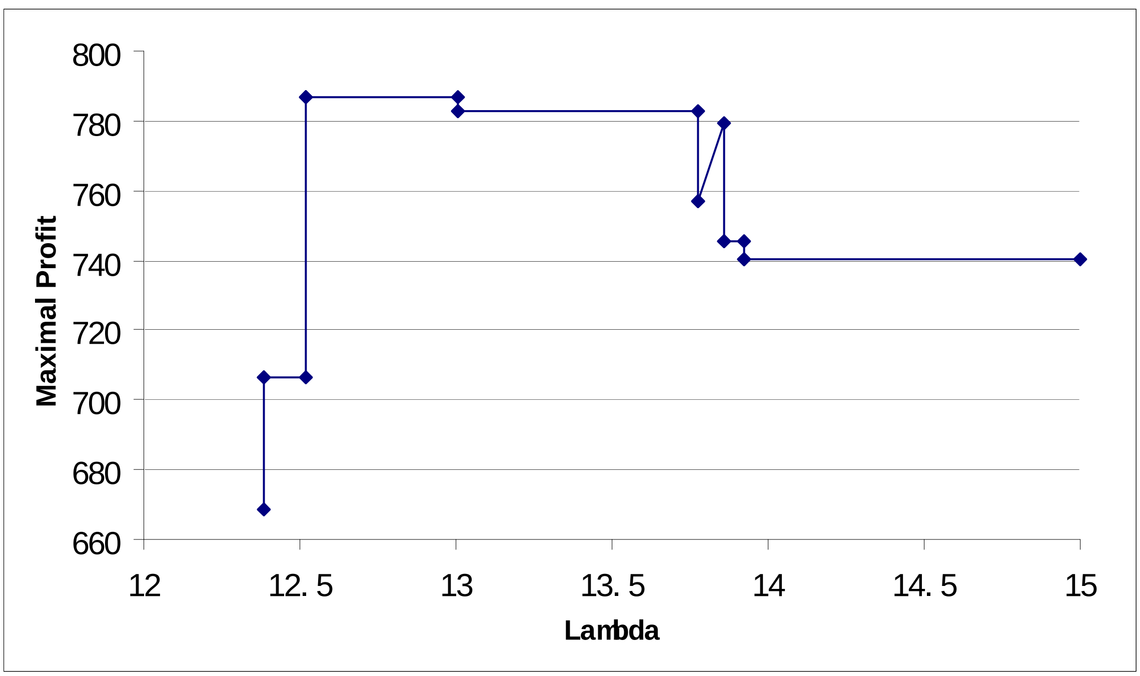

Figure 48: Company C Profit for Base Case in RTS96 System

Table 45 shows LMPs at each bus when Company C bid at his optimal strategy. It can be observed that line 3 (from Bus 1 to 5) is binding at the inverse direction, which causes LMP at Bus 5 is higher than all of others.

Table 44: Profit for Base Case in RTS96 System

Alpha	**Profit**
12.385	706.297
12.52	706.297
12.52	786.645
13.009	786.645
13.009	782.751
13.774	782.751
13.774	757.159
13.86	779.378
13.86	745.614
13.925	745.614
13.925	740.109
15	740.109

Table 45: LMP for Base Case in RTS96 System

1 14.7624	5 15.4089	9 14.0355	13 13.8543	17 13.6191	21 13.6324
2 14.704	6 14.4421	10 14.3601	14 13.2433	18 13.626	22 13.6272
3 14.0551	7 14.1978	11 13.7633	15 13.6462	19 13.6767	23 13.7742
4 14.3364	8 14.1978	12 13.9894	16 13.6044	20 13.7395	24 13.8042

Case 2

Table 46: System Demand for Case 2

Total System Demand (MW)	800

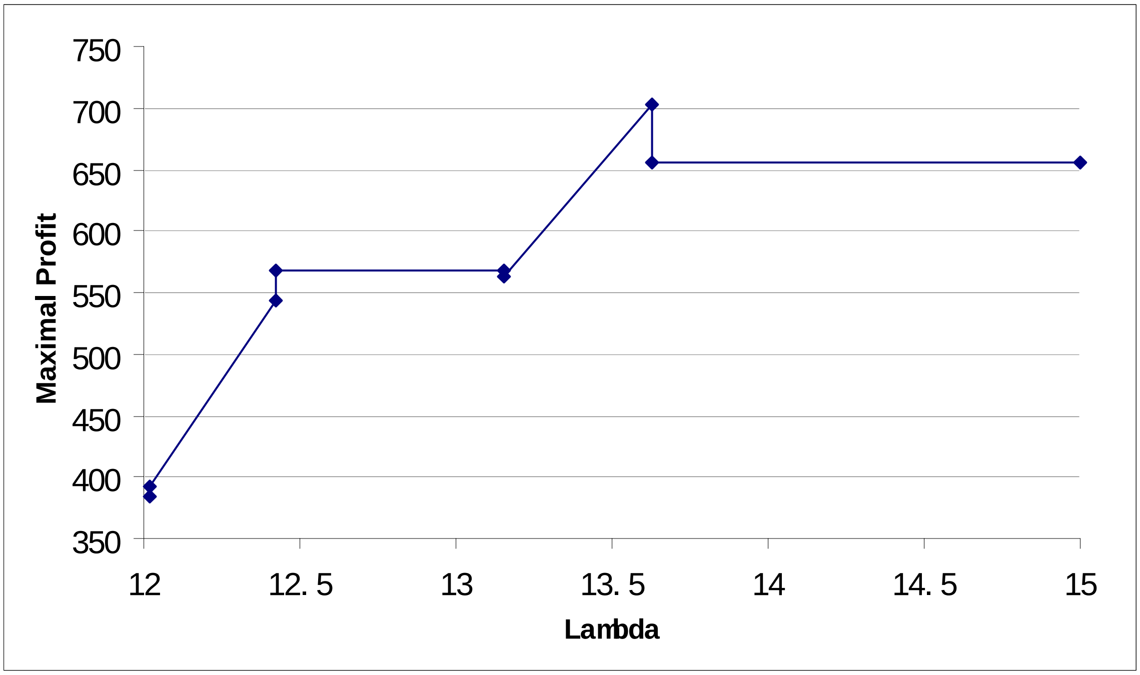

Figure 49: Company C Profit for Case 2 in RTS96 System

Table 47: Profit for Case 2 in RTS96 System

Alpha	**Profit**
12.021	383.918
12.021	392.17
12.423	543.781
12.423	567.654
13.153	567.654
13.153	563.792
13.626	702.947
13.626	655.236
15	655.236

Case 3

Table 48: System Demand for Case 3

Total System Demand (MW)	900

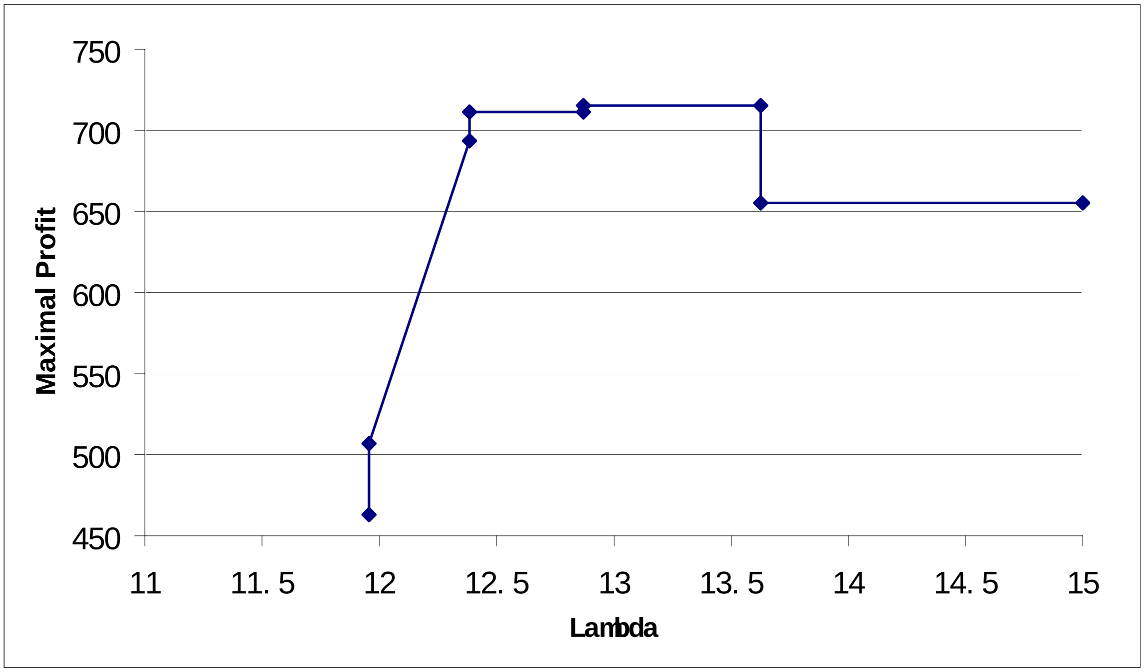

Figure 50: Company C Profit for Case 3 in RTS96 System

Table 49: Profit for Case 3 in RTS96 System

Alpha	Profit
11.955	463.126
11.955	507
12.385	693.489
12.385	711.463
12.869	711.463
12.869	715.348
13.626	715.348
13.626	655.236
15	655.236

Case 4

Table 50: System Demand for Case 4

Total System Demand (MW)	1100

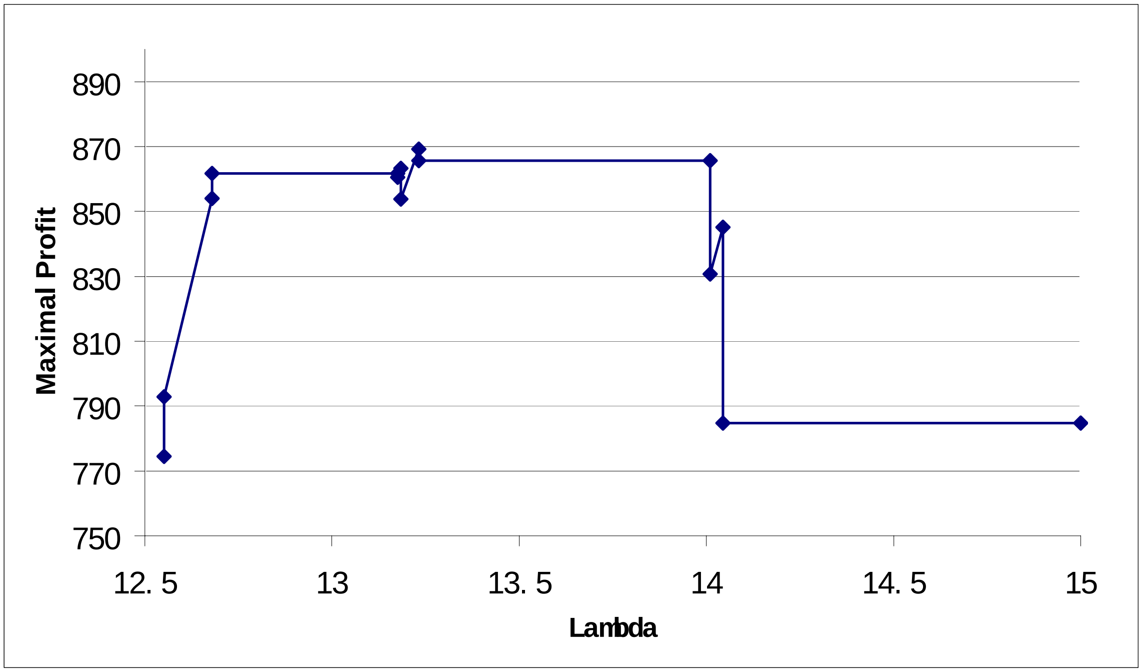

Figure 51: Company C Profit for Case 4 in RTS96 System

Table 51: Profit for Case 4 in RTS96 System

Alpha	Profit
12.551	774.513
12.551	792.846
12.68	854.025
12.68	861.662
13.176	861.662
13.176	860.502
13.184	863.358
13.184	853.855
13.232	869.136
13.232	865.678
14.01	865.68
14.01	830.64
14.044	845.204
14.044	784.664
15	784.664

Table 52: LMP for Case 4 in RTS96 System

1 15.4321	5 16.3622	9 14.386	13 14.1253	17 13.7869	21 13.806
2 15.348	6 14.9712	10 14.8532	14 13.2462	18 13.7968	22 13.7985
3 14.4143	7 14.6196	11 13.9944	15 13.8259	19 13.8698	23 14.01
4 14.8191	8 14.6196	12 14.3198	16 13.7658	20 13.9602	24 14.0533

Case 5

Table 53: System Demand for Case 5

Total System Demand (MW)	1200

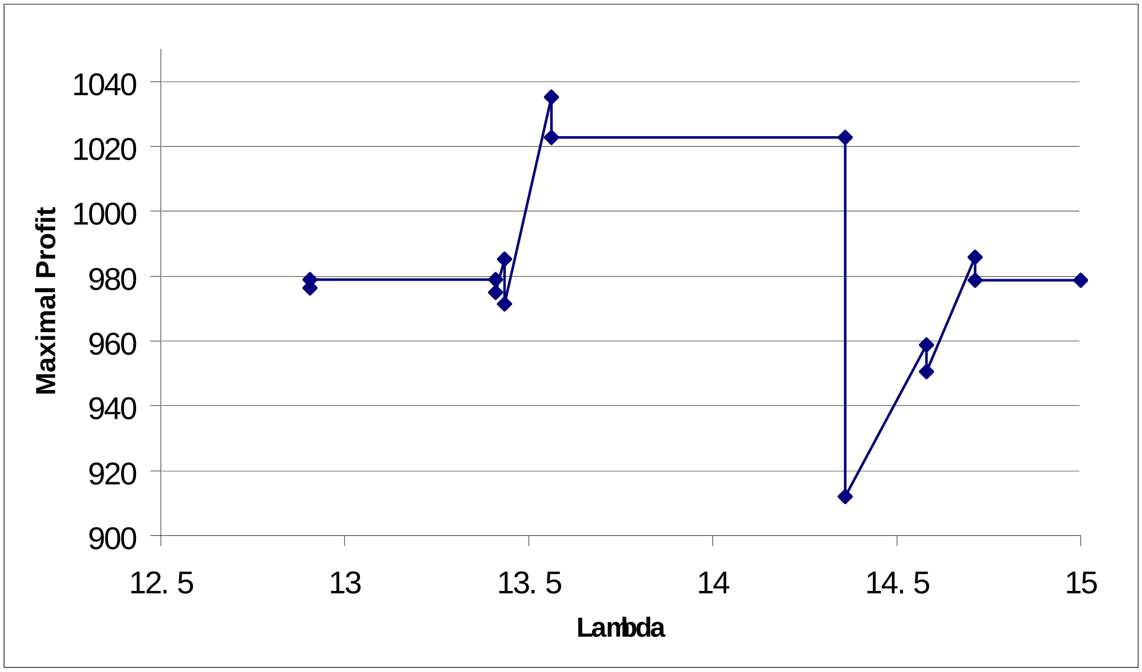

Figure 52: Company C Profit for Case 5 in RTS96 System

Table 54: Profit for Case 5 in RTS96 System

Alpha	**Profit**
12.906	976.415
12.906	979.01
13.41	979.01
13.41	975.088
13.434	985.176
13.434	971.537
13.561	1035.24
13.561	1022.83
14.359	1022.83
14.359	911.953
14.581	958.855
14.581	950.468
14.713	985.937
14.713	978.67
15	978.67

Table 55: LMP for Case 5 in RTS96 System

1 18.0426	5 20.4525	9 15.3325	13 14.657	17 13.7803	21 13.8297
2 17.8248	6 16.8486	10 16.5428	14 12.3794	18 13.806	22 13.8103
3 15.4057	7 15.9377	11 14.3178	15 13.8814	19 13.9952	23 14.3584
4 16.4546	8 15.9377	12 15.1609	16 13.7255	20 14.2293	24 14.4705

Case 6 High Bid

Table 56: System Demand for Case 6

Total System Demand (MW)	1000

Suppose all other units bid as 10% higher than marginal cost.

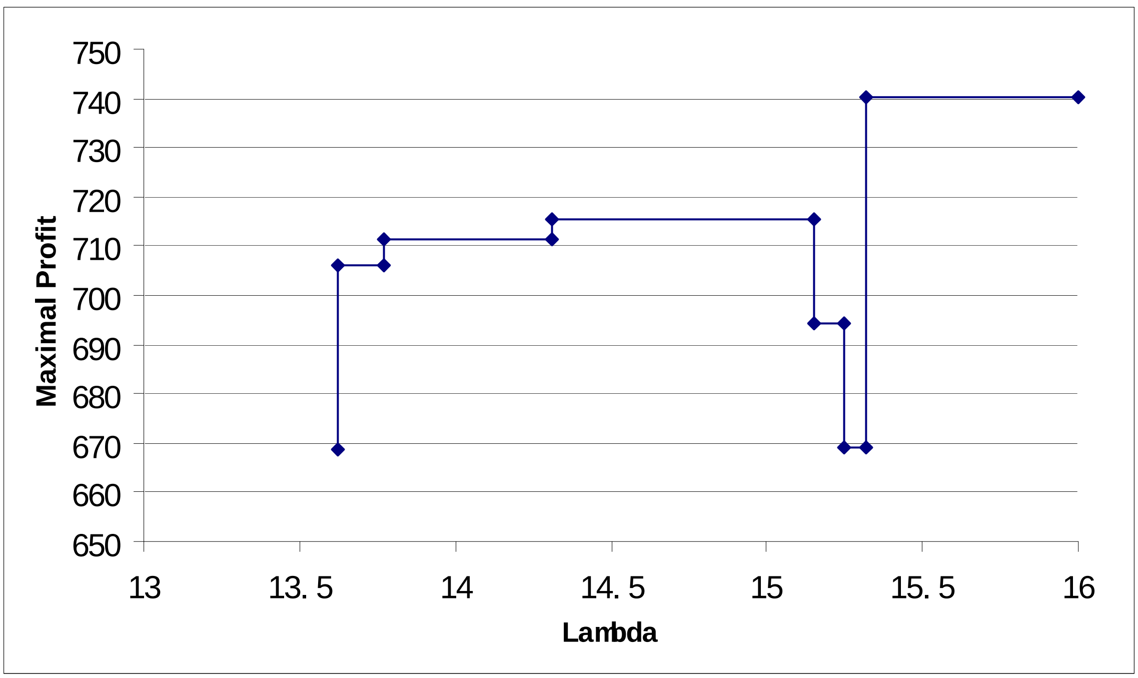

Figure 53: Company C Profit for Case 6 in RTS96 System

Table 57: LMP for Case 6 in RTS96 System

1 14.753	5 15.2946	9 14.1438	13 13.9919	17 13.7949	21 13.806
2 14.704	6 14.4846	10 14.4158	14 13.48	18 13.8007	22 13.8016
3 14.1602	7 14.2798	11 13.9157	15 13.8176	19 13.8432	23 13.9248
4 14.396	8 14.2798	12 14.1052	16 13.7826	20 13.8958	24 13.95

Table 58: Profit for Case 6 in RTS96 System

Alpha	**Profit**
13.624	668.763
13.624	706.297
13.772	706.297
13.772	711.463
14.31	711.463
14.31	715.348
15.152	715.348
15.152	694.297
15.246	694.297
15.246	669.112
15.317	669.112
15.317	740.109

Case 7 Different Bid Pattern

Table 59: System Demand for Case 7

Total System Demand (MW)	1000

Table 60: Bids of Piecewise Linear Cost Curves

Company	Unit i	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$
C	1	1.10α	1.20α	1.30α
	13	1.05α	1.15α	1.25α
	23	0.50α	0.60α	0.70α

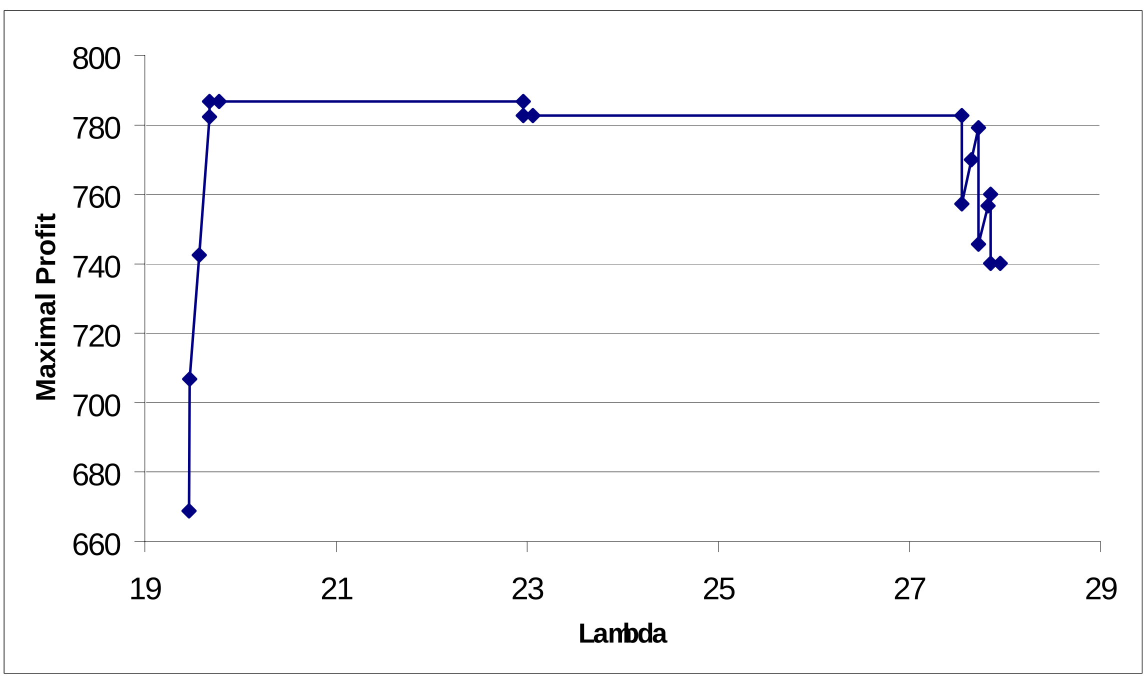

Figure 54: Company C Profit for Case 7 in RTS96 System

Table 61: Profit for Case 7 in RTS96 System

Alpha	**Profit**
19.467	706.761
19.566	742.511
19.676	782.233
19.678	786.645
19.777	786.645
22.956	786.645
22.958	782.751
23.057	782.751
27.547	782.751
27.549	757.245
27.648	770.059
27.719	779.249
27.721	745.726
27.82	756.726
27.849	759.948
27.851	740.109
27.95	740.109
30	740.109

Table 62: LMP for Case 7 in RTS96 System

1 14.7624	5 15.4089	9 14.0355	13 13.8543	17 13.6191	21 13.6324
2 14.704	6 14.4421	10 14.3601	14 13.2433	18 13.626	22 13.6272
3 14.0551	7 14.1978	11 13.7633	15 13.6462	19 13.6767	23 13.7742
4 14.3364	8 14.1978	12 13.9894	16 13.6044	20 13.7395	24 13.8042

Case 8 Incomplete Information

Table 63: Probability of Each Scenario

Scenarios	**Probability**
Base Case	0.50
High Bid	0.10
Low Demand 80%	0.05
Low Demand 90%	0.15
High Demand 110%	0.15
High Demand 120%	0.05

1. Marginal Cost Bid (12.9391)

Table 64: Profit by Marginal Cost Bid in RTS96 System

Scenarios	Profit
Base Case	786.645
High Bid	668.763
Low Demand 80%	567.654
Low Demand 90%	715.348
High Demand 110%	861.662
High Demand 120%	979.01
Expected Profit	774.0835

2. 120% Marginal Cost Bid (15.5269)

Table 65: Profit by 120% Marginal Cost Bid in RTS96 System

Scenarios	Profit
Base Case	740.109
High Bid	740.109
Low Demand 80%	655.236
Low Demand 90%	655.236
High Demand 110%	784.664
High Demand 120%	978.67
Expected Profit	741.7457

3. 110% Marginal Cost Bid (14.2330)

Table 66: Profit by 110% Marginal Cost Bid in RTS96 System

Scenarios	Profit
Base Case	740.109
High Bid	711.463
Low Demand 80%	655.236
Low Demand 90%	655.236
High Demand 110%	784.664
High Demand 120%	1022.83
Expected Profit	741.0891

4. 90% Marginal Cost Bid (11.6452)

Table 67: Profit by 90% Marginal Cost Bid in RTS96 System

Scenarios	Profit
Base Case	668.763
High Bid	668.763
Low Demand 80%	383.918
Low Demand 90%	463.126
High Demand 110%	774.513
High Demand 120%	976.415
Expected Profit	654.9203

5. Optimal Bid (13.6260)

Table 68: Profit by Optimal Bid in RTS96 System

Scenarios	Profit
Base Case	782.751
High Bid	706.297
Low Demand 80%	702.947
Low Demand 90%	715.348
High Demand 110%	865.678
High Demand 120%	1022.83
Expected Profit	785.4481

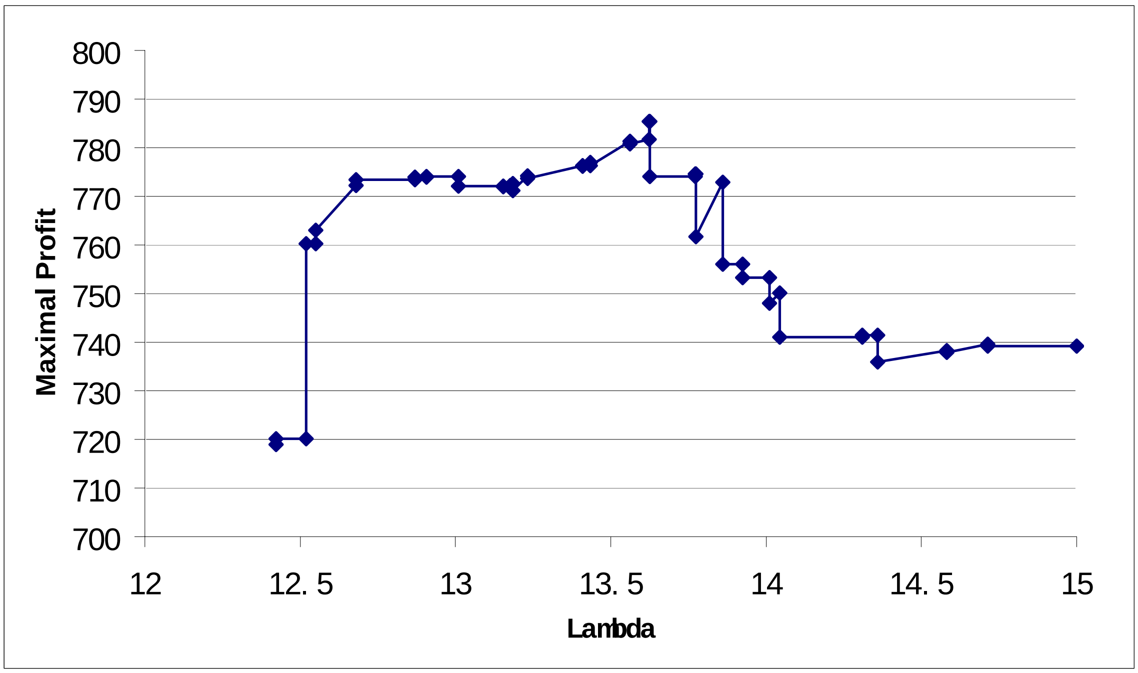

Figure 55: Company C Profit for Incomplete Information in RTS96 System

6.5 A GenBidding Tool

Fig. 56 is the Main Menu of a GenBidding tool. Users can click Optimization -> Parametric Linear Programming, which will invoke a dialog (Fig. 57). One can invoke Microsoft Excel and set up an input data file through the dialog, and start a PLP process

Table 69: Profit for Incomplete Information Case in RTS96 System

Alpha	Profit	Alpha	Profit
12.423	718.93	13.5611	780.74
12.4231	720.12	13.624	781.67
12.52	720.12	13.6241	785.42
12.5201	760.3	13.626	785.45
12.551	760.3	13.6261	774.05
12.5511	763.05	13.772	774.05
12.68	772.23	13.7721	774.56
12.6801	773.37	13.774	774.56
12.869	773.37	13.7741	761.77
12.8691	773.95	13.86	772.88
12.906	773.95	13.8601	755.99
12.9061	774.08	13.925	755.99
13.009	774.08	13.9251	753.24
13.0091	772.14	14.01	753.24
13.153	772.14	14.0101	747.99
13.1531	771.94	14.044	750.17
13.176	772.28	14.0441	741.09
13.1761	772.11	14.31	741.09
13.184	772.65	14.3101	741.48
13.1841	771.23	14.359	741.48
13.232	774.23	14.3591	735.93
13.2321	773.71	14.581	738.28
13.41	776.33	14.5811	737.86
13.4101	776.13	14.713	739.63
13.434	776.99	14.7131	739.27
13.4341	776.31	15	739.27
13.561	781.36	100	739.27

. Optimization -> Profit Maximization will invoke another dialog (Fig. 58). Profit Maximization process takes the output file of PLP as the input file and call up an optimization engine. The final maximal profit, bid, and LMP will be generated as a report. (Fig. 58)

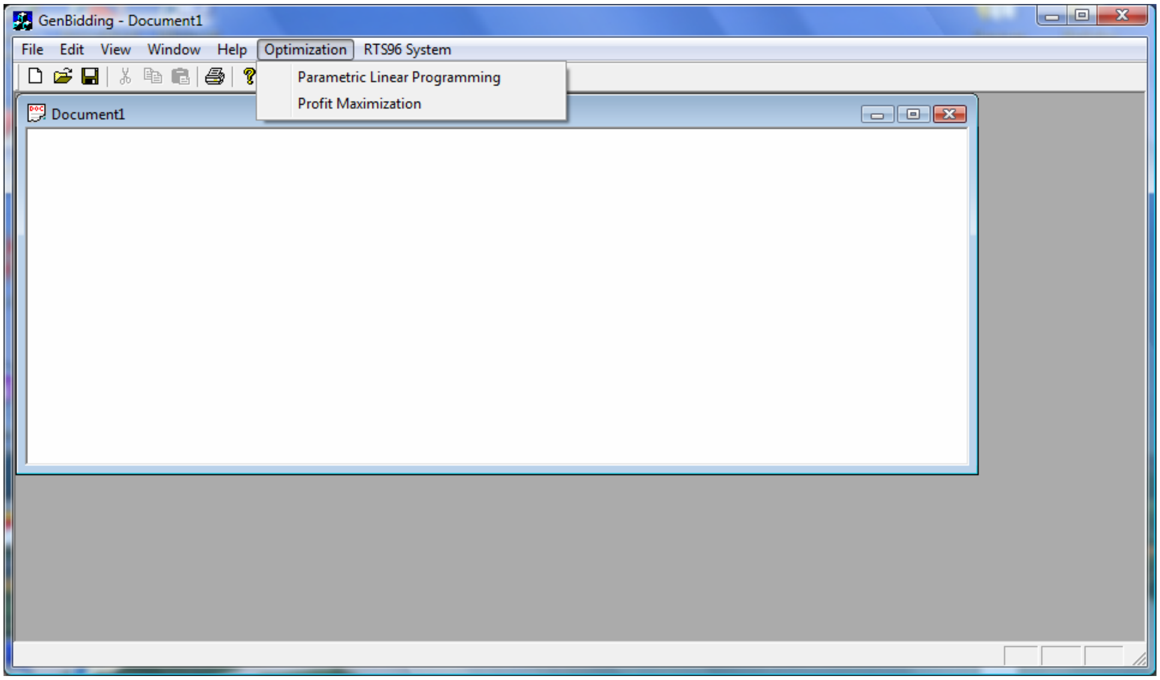

Figure 56: Main Menu of GenBidding

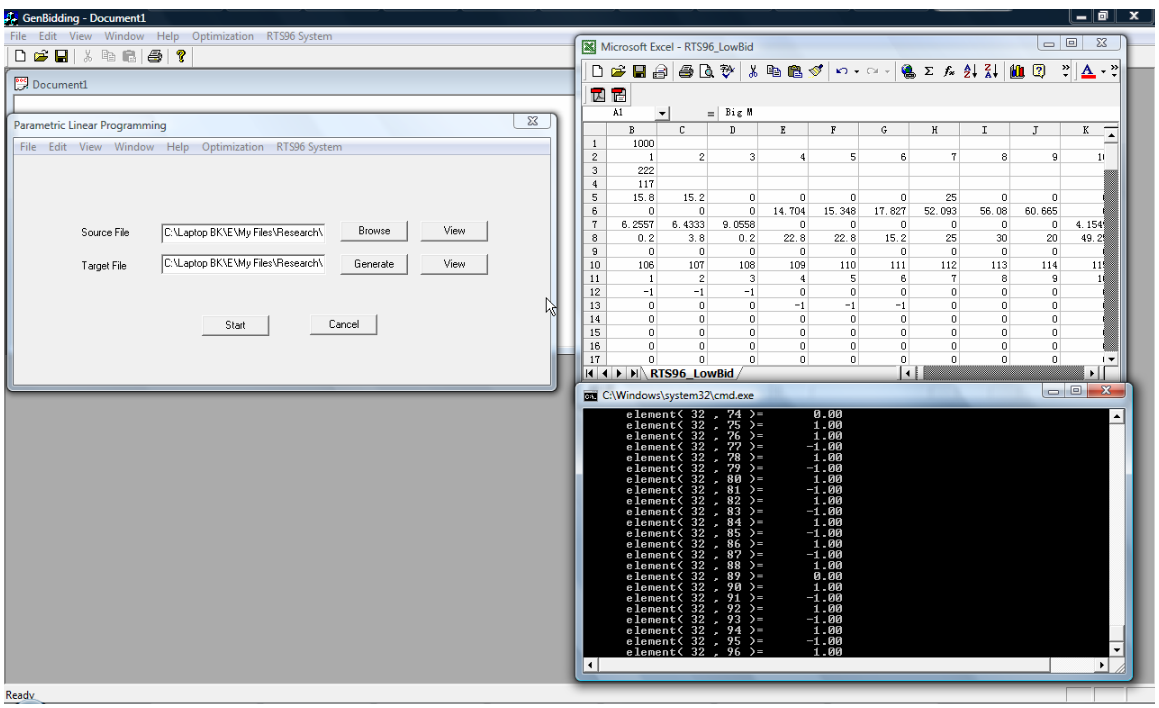

Figure 57: Parametric Linear Programming

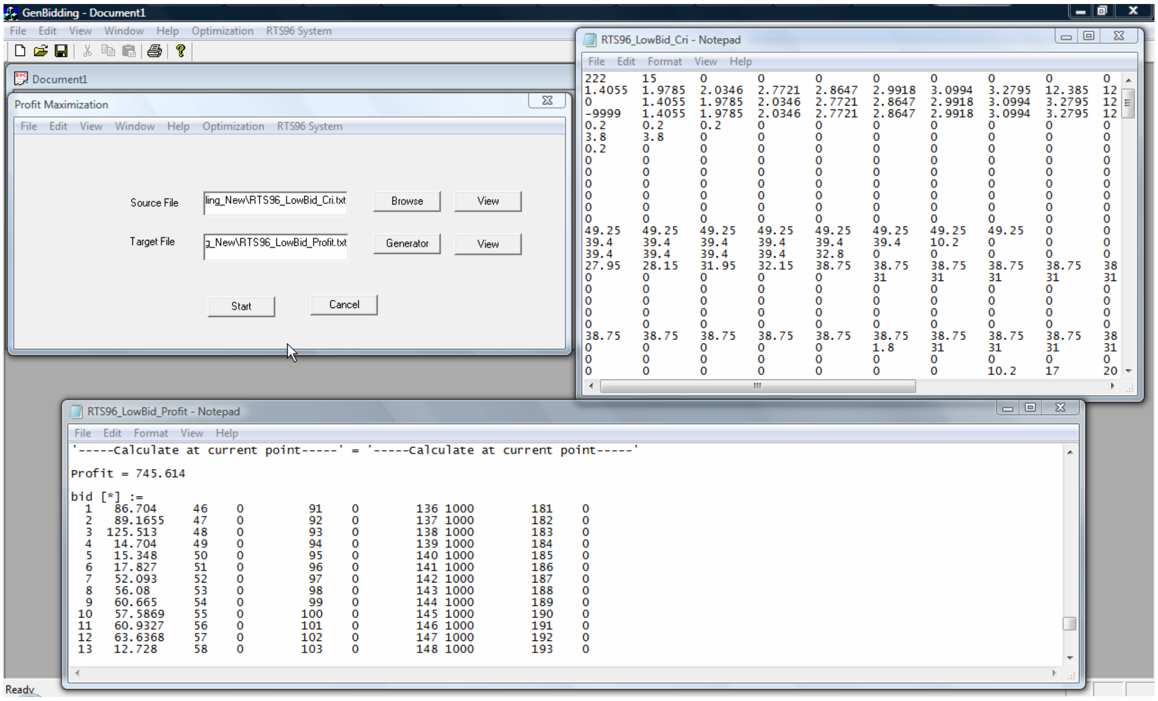

Figure 58: Profit Maximization

Users can show a system diagram (Fig. 59) through the tool.

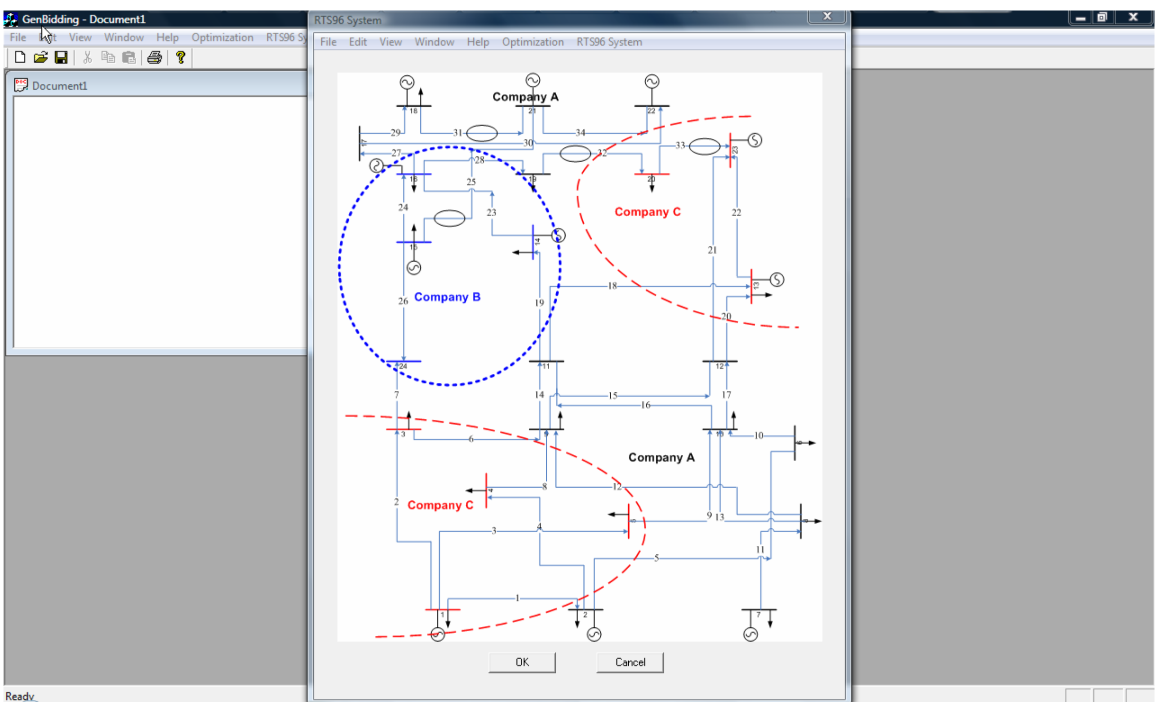

Figure 59: System Diagram

CHAPTER 7. CONCLUSION

7.1 Summary

In this dissertation, a series of novel models (one hybrid and two MILPs) are proposed for EDC with CC units. Three evolutionary algorithms (GA, EP, and PS) are implemented for EDC involving non-convex cost. A mutation prediction technique is proposed to accelerate GA. A comparison shows pros and cons of these three stochastic optimization techniques.

SFE bidding model with GENCOs' internal production constraints is developed theoretically. l_{it} – parameterization is shown to be better than k_{it} – parameterization for a multiple-period model. A discrete time optimal control problem is formulated for each GENCO. The Nash equilibrium is analytically developed by linear algebra.

SFE bidding model with transmission congestion as endogenous variables is developed theoretically. A systematic method based DC OPF sensitivity is proposed to solve for the Nash equilibria analytically. It is found that multiple equilibria may exist. This is consistent with current conjecture in the literature [24] [45].

GENCOs' bidding strategies by learning algorithms are proposed. Co-evolutionary GA [71], and Multiple population EP and PS are reviewed and designed to evolve GENCOs bidding strategies for a single-period model without transmission constraints.

GENCOs' bidding strategies based on Linear Programming are also studied. An optimal bidding strategy including Parametric Linear Programming and Linear Programming is proposed. The proposed algorithm is able to handle piecewise staircase energy offer curves, which are non-continuous and non-convex. An extension to incorporate incomplete information based on Decision Analysis is proposed. Finally, the author develops an optimal bidding tool and applies it to the RTS96 test system.

7.2 Future Extension

In the future, some possible research topics can be conducted based on this work. The author gives three aspects of possible extensions corresponding to Chapter 4,5,and 6.

1. Supple Function Equilibrium model may be extended to incorporate GENCOs' positions in a financial market. Multiple periods can be combined with transmission constraint model to investigate the theoretic market equilibria.

2. New learning algorithms considering multiple periods and transmission constraints can be developed.

3. Linear Programming bidding model is applied to a real-time market in this dissertation. The model can be extended to a day-ahead market or multiple markets (*e.g.* FTR, and Ancillary Service). Mixed Integer Programming can be applied to consider Unit Commitment schedules. Learning algorithms can combine LP model to do a complete searching.

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